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Research paper

Forecasting Returns with a Hybrid Model: Neural Network Autoregressive Market Predictions and CAPM for Asset Valuation

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Abstract:
Accurate forecasting of asset returns is essential for informed investment decisions and effective portfolio management. This paper explores a hybrid model that combines the Capital Asset Pricing Model (CAPM) with Neural Network Autoregressive (NNAR) models to enhance return predictions. While CAPM traditionally estimates expected returns based on market behavior, it has limitations due to its linear assumptions. In contrast, NNAR models excel at capturing complex, nonlinear relationships in financial time series data. Our study integrates NNAR forecasts of market returns into the CAPM framework, hypothesizing that this combined approach will yield superior accuracy, particularly in volatile market conditions. Through empirical analysis, we demonstrate that our hybrid model outperforms traditional CAPM predictions, highlighting the potential of machine learning techniques in asset valuation. The findings provide valuable insights for future research and practical applications in financial forecasting.

Keywords: CAPM; Neural Network Autoregressive, Mean square error; Asset Valuation; Financial Forecasting.
JEL Classifications: 91B64, 62P20, 97M30.

1 Introduction

Accurate forecasting of asset returns is a critical task in financial markets, driving investment decisions, portfolio management, and risk assessment. Traditional models such as the Capital Asset Pricing Model (CAPM) have been widely used to predict expected returns based on market behavior. However, CAPM relies on several assumptions, including a linear relationship between an assets returns and the markets return, which may not always hold in practice. This limitation motivates the need for more advanced techniques, such as machine learning models, that can capture the nonlinear and dynamic relationships often present in financial data.

Among these advanced techniques, Neural Network Autoregressive (NNAR) models have gained attention for their ability to forecast market returns by capturing complex patterns in time-series data. we cant forecast return by CAPM alone, so at first we forecast market return by NNAR then predict stock return by CAPM.

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By integrating NNAR-based market return forecasts into the CAPM framework, this study aims to enhance the accuracy of asset return predictions.

The objective of this paper is to examine the effectiveness of this hybrid approachusing NNAR models for forecasting market returns and applying CAPM for asset valuation. We hypothesize that this combined method will outperform traditional CAPM-based predictions, particularly in volatile market conditions. The findings of this study provide insights into the potential of machine learning techniques in improving asset pricing models.

The paper is structured as follows: Section 2 provides an overview of CAPM. Section 3 outlines neural network Autoregressive. In Section 4, we present the results and analysis. Finally, Section 5 concludes the paper with suggestions for future research.

2 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is a finance theory that establishes a relationship between the expected return of an asset and its risk, relative to the market as a whole. Developed by *William Sharpe* in 1964 [11], CAPM aims to explain how assets are priced in financial markets and helps investors determine the expected return on an asset based on its systematic risk [1, 2, 4, 8, 9].

2.1 Key Concepts

- **Risk-Free Rate** (r_f):
  The return on an asset with zero risk. Typically, government bonds of stable countries are used as a proxy for the risk-free rate.

- **Market Return** (R_M):
  The expected return of the overall market (e.g., stock market index).

- **Beta** (β):
  A measure of an asset's risk in relation to the market.

  - A **beta of 1** means the asset's price moves in line with the market.
  - A **beta greater than 1** means the asset is more volatile than the market (more risk).
  - A **beta less than 1** means the asset is less volatile than the market (less risk).

- **Market Risk Premium** ($R_M - r_f$):
  The additional return expected from investing in the market rather than a risk-free asset. This reflects the compensation investors require for taking on the risk of the market.

2.2 CAPM Formula

The CAPM equation is used to estimate the expected return of an asset:

$$E(r_i) = r_f + \beta_i \left(E(R_M) - r_f\right)$$

Where:

- $E(r_i)$ = Expected return of the asset i
- r_f = Risk-free rate
- β_i = Beta of the asset i
- $E(R_M)$ = Expected return of the market
- $E(R_M) - r_f$ = Market risk premium

2.3 Interpretation

- **Risk-Free Rate:** The minimum return an investor would expect from an absolutely safe investment.
- **Beta:** Shows how the asset moves in relation to the market. A higher beta indicates higher risk and potential reward.
- **Market Risk Premium:** Represents the extra return expected for taking on the risk of investing in the market rather than risk-free assets.

2.4 Assumptions of CAPM

CAPM makes several assumptions about the market:

- All investors are rational and risk-averse.
- There is a risk-free asset in the market.
- Investors have a one-period investment horizon.
- Markets are efficient, and all information is freely available.
- There are no transaction costs (e.g., taxes, fees).
- All investors have access to the same information and can diversify their portfolios to eliminate unsystematic risk.

2.5 Criticism of CAPM

Although CAPM is widely used, it has been criticized for several reasons:

- **Simplifying Assumptions:** Real-world markets don't always fit the assumptions of CAPM (e.g., no risk-free asset or perfect competition).
- **Betas Limitations:** The model relies heavily on beta to measure risk, but beta only captures *systematic risk* and ignores *unsystematic risk*.
- **Market Efficiency:** The assumption that markets are always efficient is not always true.
- **Empirical Evidence:** Some empirical tests of CAPM have shown that it doesnt always predict returns as accurately as claimed, leading to the development of alternative models like the *Fama-French Three-Factor Model*.

2.6 Use in Investment Decisions

Despite its criticisms, CAPM remains an important tool in finance for:

- Estimating the required return on an asset.
- Evaluating whether an asset is fairly priced.
- Assessing portfolio performance and risk-adjusted returns.

3 Neural Network Autoregressive (NNAR) Models

Neural Network Autoregressive (NNAR) models represent a powerful class of machine learning techniques used for forecasting time series data [14]. These models leverage the flexibility of artificial neural networks (ANNs) to capture complex, nonlinear relationships within time-dependent data, making them particularly well-suited for financial applications where market dynamics can be influenced by a variety of factors, including past returns, volatility, and macroeconomic conditions. In next subsections we explain more [12–15].

3.1 Artificial Neural Network

Artificial Neural Networks (ANNs) are computational models designed to mimic the structure and function of biological neural networks in the human brain. These models are widely employed to assist in various fields, including image processing, finance and economic, medical applications, quantitative forecasting, and more. In essence, ANNs consist of three primary components: inputs, processing layers, and outputs [7].

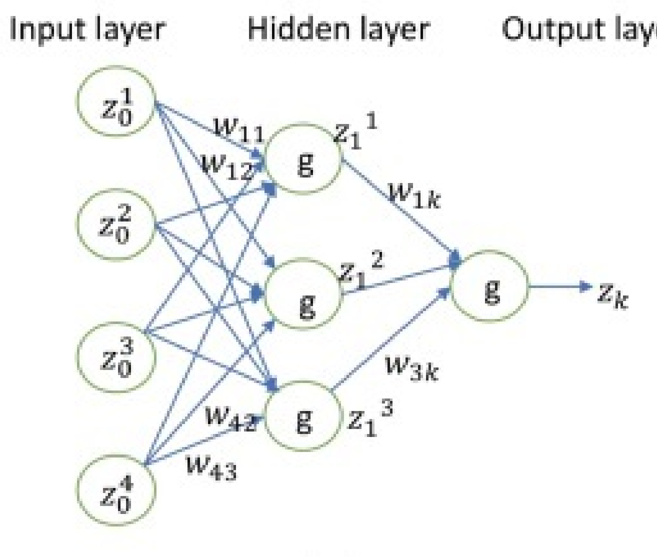

Figure 1: plot ANN with 4 input layers

The backpropagation model is an algorithmic approach in artificial neural networks that employs supervised learning [10]. This method is frequently used in ANN algorithms for forecasting models. Training or learning using backpropagation involves three main stages: feedforward propagation of the input pattern, calculation of errors from the learning process, and adjustment of the weights. In the backpropagation model, the input to each neuron is a weighted linear combination. The results of this linear combination are then transformed using a nonlinear activation function to produce the output of the ANN. The linear combination function can be expressed as:

$$z_j = g(b_j + \sum_i w_{i,j} x_i) \tag{1}$$

- z_j a variable is the sum function of the unit bias to j on the hidden layer,
- b_j a variable is a weight in the bias unit to j,
- $w_{i,j}$ a variable is the weight of the layer i bias to j,
- x_i a variable is the network input to i.

g is nonlinear binary sigmoid function that is part of the linear combination function in Equation 2. This binary sigmoid function is one of the functions for the backpropagation algorithm in a single layer network model that show in 2

$$g(x) = \frac{1}{1 + \exp(-x)} \tag{2}$$

3.2 Overview of Neural Networks and Autoregressive Models

The most common type used in time series forecasting is the feedforward neural network, which consists of an input layer, one or more hidden layers, and an output layer. These networks are trained to minimize a loss function (e.g., Mean Squared Error), adjusting the weights of the neurons to capture patterns in the input data.

In the context of time series forecasting, *autoregressive (AR)* models are often used to predict future values based on past observations. In an AR model, the value of a variable at time t is regressed on its own previous values (lags), with the assumption that past values contain important information for predicting future values. NNAR models combine the principles of autoregressive models with the flexibility of neural networks, allowing them to model complex, nonlinear dependencies between past observations and future outcomes.

3.3 NNAR for Market Return Forecasting

The Neural Network Autoregressive (NNAR) model is a type of Artificial Neural Network (ANN) where the input layer consists of a single variable with multiple lagged values, such as lag 1, lag 2, and so on, up to lag p. This model is therefore referred to as an Autoregressive ANN (NNAR). The NNAR model was introduced by Hyndman and Athanasopoulos in 2018 and is implemented in the R package "forecast" using the nnetar function. This model is specifically designed for feedforward networks with a single hidden layer and is denoted as NNAR(p, k), where p represents the number of lagged inputs and k represents the number of neurons in the hidden layer.

The NNAR method employs a single hidden layer, similar to the structure shown in Figure 1, and uses a nonlinear activation function to assign weights and produce the output from the ANN, Equation 2. The activation function used in this model is the binary sigmoid function, as described in Equation 2. For this study, the NNAR(p, k) model is implemented using the nnetar function in the R package "forecast".

One of the key advantages of NNAR over traditional linear models is its ability to model complex, nonlinear relationships. In financial markets, returns are influenced by a variety of factors, including investor sentiment, macroeconomic events, and market volatility. These factors create patterns in the data that are often non-linear and dynamic, making the assumptions of traditional models, such as CAPM, too simplistic. By using NNAR, researchers and practitioners can better capture these intricate patterns and provide more accurate forecasts of market returns.

NNAR models typically consist of an input layer, where the past returns are fed into the network, one or more hidden layers that capture complex patterns in the data, and an output layer that predicts the future market return. The number of hidden layers and neurons in each layer can be adjusted to optimize model performance.

Training the NNAR model involves using a historical dataset of market returns to adjust the weights and biases of the network. The training process typically involves backpropagation, where the model iteratively adjusts its parameters to minimize the error between predicted and actual returns. A key challenge in training neural networks is avoiding overfitting, especially when the model is highly complex or the dataset is small. Techniques such as early stopping, regularization, and cross-validation are commonly used to mitigate this risk.

Once trained, the NNAR model can be used to forecast future market returns by providing it with recent observations of market data. These forecasts can then be used as inputs into other models, such as CAPM, for more precise predictions of individual asset returns [3, 5, 6].

4 Modeling and Numerical Validations

For predicting the return of a portfolio (in this paper, we focus on a single stock in the portfolio), we follow the steps outlined below:

(i) We use NNAR with one hidden layer, 15 lags and sigmoid activation function.

(ii) We determine the best CAPM using linear regression (have maximum R-square and minimum MSE), ignoring the risk-free rate.

(iii) We forecast the market return using the best Neural Network Autoregressive (NNAR) model.

(iv) We predict the portfolio return using the market return obtained from Step 3.

(v) For CAPM we split data to train and test, 80% first data for train and 20% last data for test.

Note: For time series modeling we use forecast and for regression modeling we use predict.

4.1 Why we use this method

The CAPM model cannot directly predict future returns. To improve its accuracy, a train-test approach is typically employed. In essence, while the future remains uncertain, utilizing historical market return data as test data provides valuable insights into the potential future performance of a portfolio. To address this limitation, we incorporate NNAR (Neural Network Auto-Regressive) models.You might wonder why NNAR is not used directly to predict portfolio returns. While this was attempted, the resulting mean squared error (MSE) was higher compared to the sequential approach of first applying CAPM and then using NNAR, demonstrating that this combined methodology yields more reliable results.

4.2 Numerical results

In this paper we use the log return and adjusted close price. The formula for log return is given by:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where:

- P_t is the price at the end of the period,
- P_{t-1} is the price at the beginning of the period,
- ln is the natural logarithm.

In Table 1 we show statistical properties of stock returns.

Note: Data are daily returns, 6267 observations that driven from 2000/01/01 till 2024/11/30

stock	Mean	Variance	Min	Max	Skewness	Kurtosis
MSFT (Microsoft)	0.00039	0.00036	-0.16957	0.17869	-0.160590936	9.320113673
AAPL (Apple)	0.001	0.00055	-0.1974	0.13019	-0.18618	5.39544
AMZN (Amazon)	0.00061	0.00094	-0.28456	0.29618	0.420695812	12.9436
JNJ (Johnson & Johnson)	0.00029	0.00014	-0.17251	0.11537	-0.457671895	14.24611234
NVDA (NVDIA)	0.00117	0.00139	-0.4343	0.35357	-0.20353	12.951

Table 1: Statistical properties of Returns

In Table 2, we analyze the Capital Asset Pricing Model (CAPM) using coefficients (β_0 and β). The fourth column presents the mean squared error (MSE) of our model, which first forecasts the S&P500 returns over a 15-day period (MSE = 0.000086) and subsequently predicts the returns of our selected stocks. The last column displays the MSE for direct stock return forecasts using the Neural Network AutoRegressive (NNAR) model, indicating that our model performs better except in Apple.

In Figure 2, we present a scatter plot of market returns versus asset returns, along with the best linear regression line. Vertical axis is stock daily return and Horizontal axis is S&P 500 daily return

stock	β_0	β	MSE hybrid model	MSE NNAR	MSE CAPM
MSFT	0.00001	1.099	0.000029	0.0001	0.0002
AAPL	0.0007	1.138	0.0009	0.0008	0.0004
AMZN	0.0003	1.256	0.00025	0.001	0.0008
JNJ	0.0002	0.52	0.000036	0.0002	0.0001
NVDA	0.0008	1.653	0.0075	0.0077	0.009

Table 2: The results of CAPM, NNAR and Hybrid model

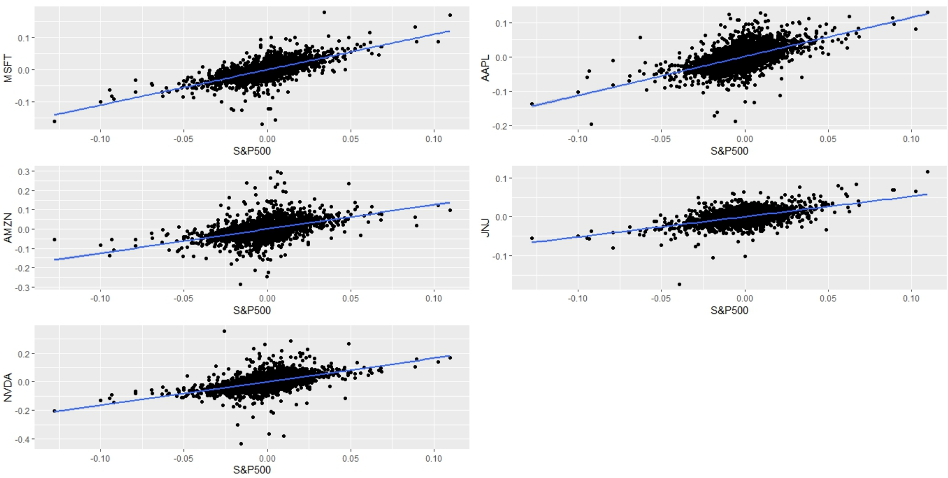

Figure 2: Plot of CAPM for all stocks

In Figure 3, we present actual vs predicted Values of stocks daily returns for CAPM model.

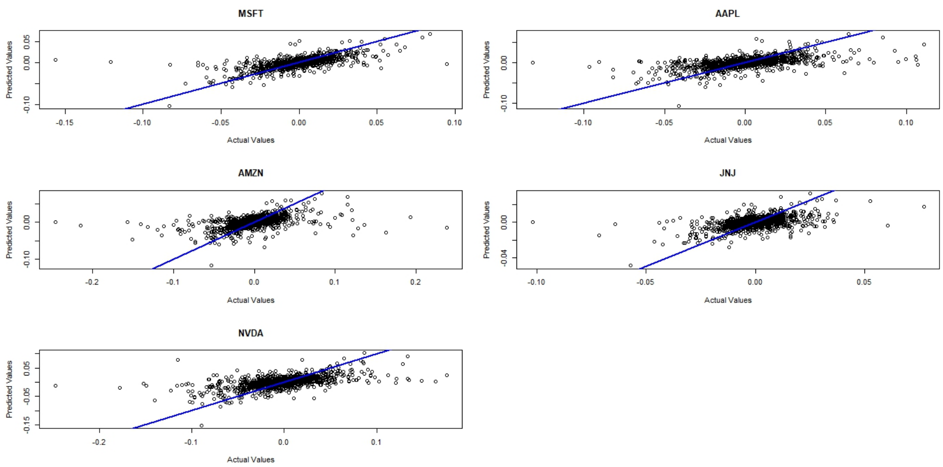

Figure 3: Plot of Actual vs Predicted Values for CAPM

5 Conclusion

In this paper, we examined the integration of Neural Network Autoregressive (NNAR) models with the Capital Asset Pricing Model (CAPM) to enhance the forecasting accuracy of asset returns. Our findings highlight the limitations of traditional CAPM, especially under volatile market conditions, where linear assumptions may fail to capture the complexity of financial data. By incorporating NNAR forecasts into the CAPM framework, we demonstrated a significant improvement in prediction accuracy.

The empirical analysis confirmed that our hybrid model outperformed traditional CAPM predictions, as evidenced by lower mean squared error values across various stocks. This suggests that machine learning techniques, particularly NNAR models, offer a powerful tool for financial forecasting, enabling a more nuanced understanding of market dynamics.

Looking ahead, future research could explore further enhancements, such as incorporating deep learning models or additional stocks into the framework. Overall, our study underscores the potential of hybrid approaches in asset valuation and encourages ongoing exploration in the field of financial forecasting.

6 Future Investigations

In future research, we can explore the following possibilities:

(i) Using Deep learning and Long short-Term Memory (LSTM) instead of NNAR

(ii) Consider including an additional stock in the portfolio.

conflict of interest

There are no relevant financial or non-financial competing interests to report.

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