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Research paper

Iran's Exchange Market in Five Episodes: Bayesian Estimation of Systematic Risk With MCMC Method

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Abstract:
This paper estimates systematic risk in Irans foreign exchange market using a stochastic volatility model, analyzing five distinct episodes shaped by varying economic and political conditions. By tracing the evolution of volatility dynamics across these episodes, we reveal critical shifts in market behavior under different risk regimes. Our results show that during low-risk episodes, volatility shocks exhibit high persistence, causing market disturbances to linger. In contrast, as systematic risk intensifies, volatility shocks dissipate more rapidlyyet this reduced persistence coincides with a marked rise in average volatility. We identify three particularly turbulent episodes in the past seven years, each characterized by exceptionally high levels of systematic risk. Strikingly, both the mean and variance of volatility increased during these high-risk periods, signaling not only heightened instability but also deeper Knightian uncertainty. These findings carry significant policy implications: when direct reduction of volatility proves challenging, policymakers should prioritize reducing the volatility of volatility to mitigate uncertainty and stabilize expectations. Notably, our analysis indicates that a 1% reduction in volatility corresponds to a 1.7% decline in the variance of daily exchange rate returns, underscoring the leverage policymakers have over market uncertainty.

Keywords: Exchange Rate, Stochastic Volatility model, MCMC Method
Classification: C11, C58, F31, F47.

1 Introduction

The exchange rate is a key and important variable in understanding the macroeconomic situation. In an economy where a large part of its exports is oil revenues, the exchange rate can experience very large fluctuations. Exchange rate fluctuations have the ability to be transmitted to the economy and, as a result, can affect many economic variables. On the other hand, many shocks that enter the economy show their effect on the exchange rate. For example, high inflation ultimately affects foreign goods and causes an increase in the exchange rate. Therefore, it can be claimed that the exchange rate is one of the most important economic variables to

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explain macroeconomic risks.

Systematic risk refers to the risk that the entire economy is exposed to and therefore cannot be diversified. The role of the policymaker in reducing systematic risk is very key. The aim of this research is to measure systematic risk through exchange rate fluctuations for Iran in different cycles. We show that external shocks strongly affect systematic risks and play a serious role in shaping the systematic risks of the Iranian economy. Despite the fact that the policymaker faces serious limitations, he still has tools through which he can manage systematic risk. We define the concept of volatility and show that if the central bank targets this variable, it will be able to control part of the exchange rate risk. From a statistical perspective, we use a new method for performing Monte Carlo simulations, called Hamiltonian Monte Carlo. This method has several advantages, which will be explained below.

With this assumption, we try to extract systematic risk from the exchange rate using stochastic volatility models. In such models, volatility is assumed to be a hidden variable that must be estimated with the help of the model. Stochastic volatility models - as their name suggests - are used to estimate stochastic volatility. These models are visually appealing because they estimate volatility over time. However, since the likelihood function is difficult to evaluate, it is complicated to estimate. These models are usually applied to stock indices or currency pairs and estimate the volatility of that index.

We first divide the exchange rate situation into five episodes based on the exchange rate distribution. In fact, each episode has its own dynamics. Next, we try to estimate the parameters related to each episode using stochastic volatility models. Finally, we analyze each section and examine the volatility of each episode, which indicates the systematic risk that the country has incurred in that period. We express the stochasticity of volatility in terms of a class of stochastic differential equations. Stochastic differential equations are suitable for high-frequency data, and we also use daily data in our estimation.

The continuation of the path is as follows: In Section 2, a review of the literature on stochastic volatility models will be provided, and empirical work related to Iran will also be reviewed. Section 3 will be related to the explanation of the model. Section 4 is dedicated to describing the estimation method. In Section 5, the division of different exchange rate periods in Iran will be explained, and in Section 6, the results will be analyzed.

2 Literature Review

A major application of stochastic volatility models is in financial modeling, and in particular asset pricing (see Barndorff-Nielsen et al. [2] for an example). Taylor [31] first proposed that the logarithm of volatility can be modeled as a first-order autoregressive process (AR(1)). The details of the model will be discussed later. Nelson [26] has introduced three fundamental drawbacks to the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) family of models in the field of asset pricing: (1) Researchers, starting with Black [27], have found a negative correlation between current returns and future return volatility. GARCH models reject this hypothesis. (2) GARCH models impose parameter constraints that are often violated by the estimated coefficients and may inappropriately constrain the dynamics of the conditional variance process. (3) It is difficult to interpret whether conditional variance shocks are persistent in GARCH models because the usual norms for measuring persistence often do not match. Stochastic volatility models have overcome these drawbacks.

Various methods have been proposed to estimate the parameters of this model. Melino and Turnbull [24] used the GMM method. Duffie and Singleton [7] estimated the parameters using the method of moment simulation, using the moments of a simulated process. A new method was developed by Jacquier et al. [16]. They estimated the parameters using Markov Chain Monte Carlo (MCMC). For more details and other methods, see Broto and Ruiz [5].

The logarithmic volatility model is a very common model, but it has its own problems. First, volatility is constant, which is not compatible with the real world. Second, numerical computation of option prices is very expensive. To address these concerns, alternative models have been developed. Heston introduced the square root stochastic volatility model. In this model, volatility includes the square root of volatility. After him, Bates [3,4], Pan [28], and Duffie et al. [8] extended this model and added jumps to returns and volatility. Eraker et al. [10] estimated stochastic volatility models with jumps in returns and volatility using MCMC, and Eraker [9] extended it by including option prices.

The research conducted on Iran is divided into two main categories. The first category is research that has used stochastic volatility models to estimate the parameters of the Iranian stock market returns. Heybati et al. [13] have shown that the uncertainty estimate is strongly influenced by the predictive regression equations and finally concludes that the autoclastic volatility model and asymmetric GARCH have better forecasting in and out of the sample. Amiri [1] has fitted five types of volatility models, namely ARCH, GARCH, SV-AR, SV-STAR, and SV-MSAR, to the Iranian stock market data set in a Bayesian framework using MCMC methods. The results of his studies show that SV models perform better than ARCH and GARCH models. Momenzadeh et al. [25], using two stochastic volatility and stochastic volatility models, reject the hypothesis of the existence of a unit root in the Iranian stock market between 2019-2021. Of course, they conclude that the stability of volatility in the Iranian stock market is very high.

The second category is research conducted on exchange rates. Dargahi and Ansari [6] have improved the neural network model for forecasting exchange rates based on the variance volatility index. For this purpose, they have considered the two variance indices and GARCH as exchange rate volatility indices separately and have used them in the model in two ways. First, they added its lag to the exchange rate lags, and second, they leveled the volatility index and, by classifying observations based on the level of volatility, used a specific forecasting model for each category of observations. The results show that models with high levels of volatility, compared to the baseline model, improve the forecasting power of future exchange rates. Tayebi et al. [30], in a study based on neural networks, investigated the hypothesis that the neural network model performed better than other models, and the results indicated the correctness of this hypothesis. Bafandeh Imandoust et al. [15], in a study on exchange rate forecasting based on ARIMA, fuzzy neural network, and autoregressive neural network models, dealt with the results. The results of comparing the three models mentioned based on different forecasting criteria show that in exchange rate forecasting, the fuzzy neural network model is superior to competing models. The first study that used stochastic differential equations to forecast exchange rates is the article by Khodavaisi and Molabahrami [20], who showed that the geometric Brownian motion model has better forecasting power than the ARIMA model.

Therefore, this research is noteworthy in several ways. First, the Iranian foreign exchange market has not been analyzed and studied within the framework of stochastic volatility models. Second, our analysis tool is the Hamiltonian Monte Carlo method, which has high accuracy in estimation and will be discussed below. The third point is that in our analysis, we divide the foreign exchange market in Iran into five episodes and assume that the dynamics of each episode are different from the rest, which allows us to extract very interesting intuitions.

3 Model

We use a widely used stochastic volatility model in which volatility follows an AR(1) form. According to Johannes and Polson [18],

$$\begin{aligned} \log(S_t) &= \mu\, dt + \sqrt{V_t}\, W_t^s \\ \log(V_t) &= \kappa_v\, (\theta_v - \log(V_t))\, t + \sigma_v\, W_t^v \end{aligned}$$

In our model, there are two stochastic differential equations, one describing the dynamics of the asset price (here the exchange rate) and one describing the dynamics of the asset price volatility. In this model, S_t is the exchange rate and V_t is the squared volatility of the exchange rate, and κ_v, θ_v, σ_v, and μ are the parameters of the problem. For simplicity, we assume that the Brownian motions of the exchange rate and volatility are independent, although Jacquier et al. [17] have relaxed this assumption and introduced the leverage effect into the model.Using Euler's method, the above differential equations can be written in discrete form

$$\begin{aligned} Y_t &= \mu + \sqrt{V_{t-1}}\, \varepsilon_t^s \\ \log(V_t) &= \alpha_v + \beta_v \log(V_{t-1}) + \sigma_v \varepsilon_t^v, \end{aligned}$$

that ε_t^s and ε_t^v are two independent shocks with $\varepsilon_t^s, \varepsilon_t^v \sim \mathcal{N}(0,1)$. Also, according to the daily exchange rate return data in Iran, the μ can be considered zero and eliminated.

$$\alpha_v = \kappa_v \theta_v, \qquad \beta_v = 1 - \kappa_v.$$

If we assume the priors for the parameters of the conjugate model as $\alpha_v, \beta_v \sim \mathcal{N}$ and $\sigma_v^2 \sim \mathcal{IG}$, we will have

$$p(\alpha_v, \beta_v | \sigma_v, V, Y) \propto \prod_{t=1}^{T} p(V_t | V_{t-1}, \alpha_v, \beta_v, \sigma_v)\, p(\alpha_v, \beta_v) \sim \mathcal{N}$$
$$p(\sigma_v^2 | \alpha_v, \beta_v, V, Y) \propto \prod_{t=1}^{T} p(V_t | V_{t-1}, \alpha_v, \beta_v, \sigma_v)\, p(\sigma_v^2) \sim \mathcal{IG},$$

that $\mathcal{N}$ and $\mathcal{IG}$ refer to Normal and Inverse-Gamma distributions. It can also be shown using Bayes' rule that

$$\begin{aligned} p\left(V_t | V_{t-1}, V_{t+1}, \Theta, Y\right) &\propto p\left(V_t, V_{t-1}, V_{t+1} | \Theta, Y\right) \\ &\propto p\left(Y_t | V_t, \Theta\right) p\left(V_{t-1}, V_t, V_{t+1} | \Theta\right) \\ &\propto p\left(Y_t | V_t, \Theta\right) p\left(V_t | V_{t-1}, \Theta\right) p\left(V_{t+1} | V_t, \Theta\right)) \end{aligned}$$

Therefore, the model parameters can be estimated using the Gibbs sampling method, as well as the latent variable using the Metropolis-Hasting algorithm, which is described in more detail in Johannes and Polson [18].

4 Estimation Method

In this section, the Markov Chain Monte Carlo implementation method is introduced and its theoretical basis will be explained. The method we use to estimate the model is much more efficient, albeit computationally intensive, than traditional methods. The Hamiltonian Monte Carlo (HMC) algorithm, also called hybrid Monte Carlo and have been developed recently, was developed by Duane in physics first. It moves more quickly towards the target distribution by suppressing the random walk behavior of the Metropolis-Hastings algorithm. For each component θ_j in the target space, the Hamiltonian Monte Carlo adds a momentum variable ϕ_j. Both θ and ϕ are then updated together in a new Metropolis algorithm, in which the jump distribution for θ is largely determined by ϕ.

According to Gelman et al. [12], HMC proceeds by a series of iterations (as in any Metropolis algorithm), with each iteration having three parts:

(i) The iteration begins by updating with a random draw from its posterior distribution which, as specified, is the same as its prior distribution, $\phi \sim \mathcal{N}(0, M)$.

(ii) The main part of the Hamiltonian Monte Carlo iteration is a simultaneous update of (θ, ϕ), conducted in an elaborate but effective fashion via a discrete mimicking of physical dynamics. This update involves L leapfrog steps (to be defined in a moment), each scaled by a factor ϵ. In a leapfrog step, both θ and ϕ are changed, each in relation to the other. The L leapfrog steps proceed as follows:

Repeat the following steps L times:

(a) Use the gradient (the vector derivative) of the log-posterior density of θ to make a half-step of ϕ:

$$\phi \leftarrow \phi + \frac{1}{2}\epsilon \frac{\log p(\theta|y)}{\theta}$$

(b) Use the momentum vector ϕ to update the position vector θ:

$$\theta \leftarrow \theta + \epsilon M^{-1}\phi$$

(c) Again use the gradient of θ to half-update ϕ:

$$\phi \leftarrow \phi + \frac{1}{2}\epsilon \frac{\log p(\theta|y)}{\theta}$$

(iii) Label θ^{t-1}, ϕ^{t-1} as the value of the parameter and momentum vectors at the start of the leapfrog process and θ^*, ϕ^* as the value after the L steps. In the accept-reject step, we compute

(iv) Set

$$\theta^t = \begin{cases} \theta^* & \text{with probability } \min(r, 1) \\ \theta^{t-1} & \text{otherwise.} \end{cases}$$

Probabilistic programming (PP) allows flexible specification of Bayesian statistical models in code. PyMC is a PP framework with an intuitive and readable, yet powerful, syntax that is close to the natural syntax statisticians use to describe models. It features next-generation Markov chain Monte Carlo (MCMC) sampling algorithms such as the No-U-Turn Sampler (NUTS; see Hoffman et al. [14]), a self-tuning variant of Hamiltonian Monte Carlo. We use the PyMC package in Python, which allows us to achieve high accuracy in Bayesian estimation. For more details about this package, see Salvatier et al. [29].

5 Data Preparation

We use daily data on the Iranian exchange rate between 09/07/2008 and 10/04/2025. If we examine the histogram of this data (Figure 1), it is observed that there are approximately 5 peaks in it. In other words, the exchange rate has fluctuated around

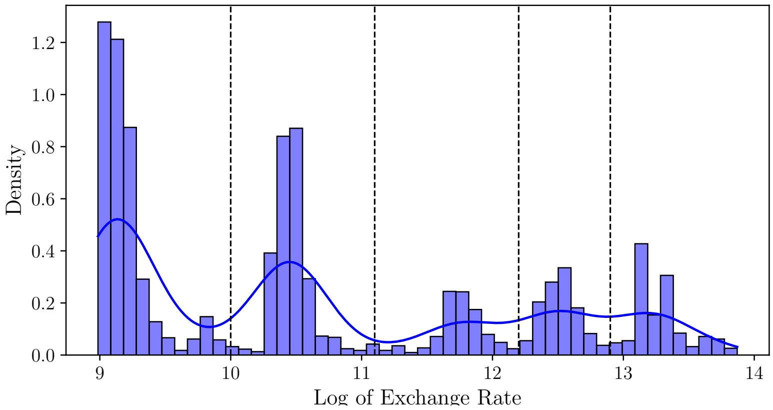

Figure 1: Histogram of Logarithm of Exchange Rate

5 peaks, which indicates that the exchange rate has been in 5 different regimes, and naturally, each regime has its own dynamics.

Now, if we combine the time series with the histogram and find the time period of each episode, we get 5 time periods that correspond in an interesting way to political events and major systemic risks in Iran. The first episode is related to the abundance of oil revenues and the first period of sanctions. In the second episode, with the JCPOA agreement, systemic risks have decreased and more calm prevails. The third episode shows the second round of sanctions, which was also the most severe period of sanctions. In the fourth episode, pressures and also systemic risks decrease, and the fifth episode coincides with the second term of Trumps presidency. A summary of this division is given in Table 1.

Figure 2: Histogram with Time series of Logarithm of Exchange Rate

Episode	Description	Period
1	High oil income and first period od sanction	2008/07/09 – 2012/09/02
2	JCPOA agreement and low systematic risk	2012/09/03 – 2018/06/19
3	Most severe period of sanction	2018/06/20 – 2020/06/22
4	Decreasing pressure and systematic risk	2020/06/23 – 2023/02/18
5	Second term of Trumps presidency	2023/02/19 – 2025/04/10

Table 1: Episodes of Exchange Rate Regimes

Table 2 presents the descriptive statistics of the daily returns of the episodes. Column (1) indicates that the research encompasses a total of observations. Columns

(2) to (3) present the mean and standard deviation, respectively, for the entire episode. Moving on to Columns (4) to (8), they provide information on the minimum value, the 25th percentile, the 75th percentile, and the maximum value of the returns. The difference in standard deviation is quite evident in episodes 3 and 4. Results are in percentages.

Variable	**N**	**Mean**	**Std**	**Min**	**P25**	**P75**	**Max**
Episode 1	1069	0.085	1.120	-10.53	-0.190	0.296	10.53
Episode 2	1654	0.080	1.215	-9.91	-0.192	0.285	17.09
Episode 3	584	0.1978	2.665	-20.27	-0.648	0.933	16.12
Episode 4	785	0.1265	1.627	-10.86	-0.411	0.731	12.76
Episode 5	729	0.1063	1.200	-7.97	-0.400	0.586	7.69
Total	4825	0.1086	1.516	-20.27	-0.278	0.428	17.09

Table 2: Descriptive Statistics

To further confirm the differences between the periods, we looked at the distribution of daily exchange rate returns in each episode (Figure 3). Although the mean returns are almost the same, the variances are quite different. Episodes 1 and 2 have similar distributions, but the variances of the distributions for episodes 3 to 5 are much higher. In particular, episode 3, which was the most difficult economic situation, has a distribution with very fat tails, indicating that there were many days with high volatility. Therefore, we can conclude that there were 5 different periods in the Iranian economy.

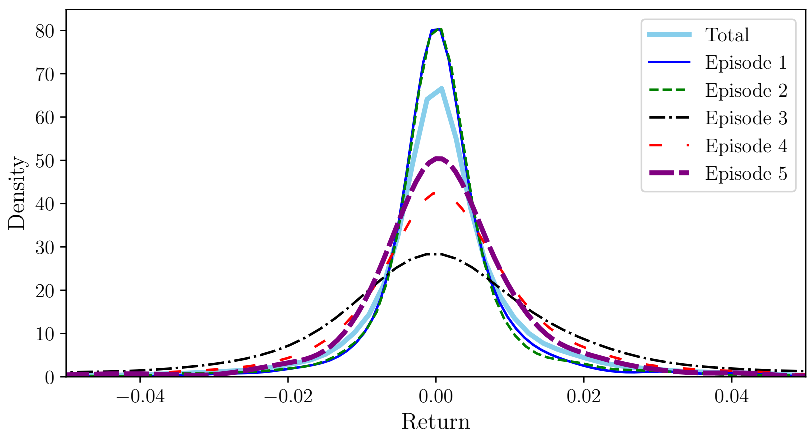

Figure 3: Distribution of Daily Return of Episode

param	mean	sd	hdi 3%	hdi 97%	mcse mean	mcse sd	ess bulk	ess tail	r hat
α_v	-0.041	0.030	-0.098	0.016	0.0	0.0	19672.0	21144.0	1.0
β_v	0.996	0.003	0.991	1.001	0.0	0.0	19715.0	20820.0	1.0
σ_v	0.157	0.013	0.134	0.183	0.0	0.0	1421.0	3087.0	1.0

Table 3: Parameters of Episode 1

param	mean	sd	hdi 3%	hdi 97%	mcse mean	mcse sd	ess bulk	ess tail	r hat
α_v	-0.047	0.021	-0.087	-0.008	0.000	0.000	45238.0	27259.0	1.0
β_v	0.996	0.002	0.992	0.999	0.000	0.000	46206.0	26919.0	1.0
σ_v	0.132	0.006	0.120	0.144	0.000	0.000	2274.0	5389.0	1.0

Table 4: Parameters of Episode 2

param	mean	sd	hdi 3%	hdi 97%	mcse mean	mcse sd	ess bulk	ess tail	r hat
α_v	-0.208	0.076	-0.353	-0.067	0.000	0.0	37699.0	30035.0	1.0
β_v	0.975	0.009	0.959	0.992	0.000	0.0	34869.0	30314.0	1.0
σ_v	0.533	0.050	0.440	0.627	0.001	0.0	2464.0	4501.0	1.0

Table 5: Parameters of Episode 3

param	mean	sd	hdi 3%	hdi 97%	mcse mean	mcse sd	ess bulk	ess tail	r hat
α_v	-0.233	0.072	-0.368	-0.098	0.000	0.0	38859.0	28232.0	1.0
β_v	0.974	0.008	0.959	0.989	0.000	0.0	38147.0	28734.0	1.0
σ_v	0.359	0.029	0.305	0.414	0.001	0.0	2467.0	4997.0	1.0

Table 6: Parameters of Episode 4

param	mean	sd	hdi 3%	hdi 97%	mcse mean	mcse sd	ess bulk	ess tail	r hat
α_v	-0.173	0.065	-0.294	-0.049	0.000	0.0	25577.0	26028.0	1.0
β_v	0.982	0.007	0.969	0.995	0.000	0.0	25705.0	27794.0	1.0
σ_v	0.247	0.025	0.202	0.294	0.001	0.0	1602.0	3246.0	1.0

Table 7: Parameters of Episode 5

6 Result

We used four chains for sampling, each chain containing 5000 samples for tuning and 10000 samples for estimation. Tables 3 to 7 show the estimation of the problem parameters. We will use the mean of the distribution to estimate the point parameters of the model. The α_v parameter or intercept is negative in all episodes. As is clear from Table 8, the negative value is larger in periods with higher systematic risk. This means that the system tries to change its dynamics in a way that controls volatility.

param	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
α_v	-0.041	-0.047	-0.208	-0.233	-0.173
standard deviation	0.030	0.021	0.076	0.072	0.065

Table 8: Intercept in Episodes

Table 9 shows the estimate of the β_v parameter. This coefficient shows the persistence of a shock to volatility. In periods when the economy is in a calm state, this value is very high. This means that a shock to volatility remains for a long time (its half-life is more than 100 periods), but in periods with high systematic risk β_v is decreased. However, since α_v is negative, a lower β_v means higher volatility because the average logarithm of volatility in the stationary state follows equation 1. So, it is true that with the lower β_v, shocks are damped faster, but the average volatility will also increase. Of course, each shock still has a half-life of about 25 periods, which is relatively high.

$$[\log V_t] = \frac{\alpha_v}{1-\beta_v} \tag{1}$$

param	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
β_v	0.996	0.996	0.975	0.974	0.982
standard deviation	0.003	0.002	0.009	0.008	0.007

Table 9: Persistence in Episodes

However, the last variable, which is very important, is σ_v, which indicates the standard deviation of volatility shocks. In fact, its large size means an increase in systematic risk to the economy. In the third episode, which is the most difficult economic period, its value is almost three times that of the previous period. However, after that, the economy has been able to reduce it over time. Of course, its value is still far from the first and second episodes.

param	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
σ_v	0.157	0.132	0.533	0.359	0.247
standard deviation	0.013	0.006	0.050	0.029	0.025

Table 10: Volatility in Episodes

6.1 Convergence Diagnosis

In the MCMC method, an important issue that must be addressed is the convergence of estimates. To achieve this goal, we will use three indicators: the effective sample size, the Goleman-Robin statistic, and the Monte Carlo standard error.

Effective Sample Size

When using MCMC sampling methods, it is reasonable to ask whether a particular sample is large enough to reliably calculate the values of interest, such as the mean.

This is something we cannot answer directly by just looking at the number of samples, because the samples from MCMC methods will be somewhat autocorrelated, so the actual amount of information contained in that sample will be less than the information we would get from an iid sample of the same size. We say that a series of values is autocorrelated when we can observe a similarity between them as a function of the time interval between them.

According to Kass et al. [19] and Liu and Liu [22], the effective sample size (ESS) index is defined as follows

$$ESS = \frac{n}{1 + 2\sum_{k=1}^{\infty} \rho_k(\theta)},$$

where n is the total sample size and $\rho_k(\theta)$ is the lag-k autocorrelation for θ. In Tables 3 to 7, two indicators for ESS are reported. One of them is **bulk-ESS** which mainly assesses how well the center of the distribution was resolved. If you also want to report posterior intervals or you are interested in rare events, you should check the value of **tail-ESS**, which corresponds to the minimum ESS at the percentiles 5 and 95 (see Martin et al. [23]).

According Vehtari et al. [32], as a general rule of thumb we recommend a value of ESS greater than 400, otherwise, the estimation of the ESS itself and the estimation of other quantities will be basically unreliable. In all parameters and in all episodes, this value is at least 1400, indicating that the estimate is acceptable.

Gelman-Rubin Statistic

Under very general conditions, Markov chain Monte Carlo methods have theoretical guarantees that they will arrive at the correct answer regardless of the starting point. Unfortunately, as explained, these guarantees are only valid for infinite samples. So in practice we need methods to estimate convergence for finite samples. A general idea is to run more than one chain, starting from very different points, and then examine the resulting chains to see how similar they are. This intuitive concept can be formulated as a numerical test called Gelman-Rubin statistic ($\hat{R}$).

There are many versions of this estimator, as it has been refined over the years. According to Gelman and Rubin [11], J Monte Carlo simulations (chains) are started with different initial values. The samples from the respective burn-in phases are discarded. The mean of the means of all chains is

$$\bar{x}_* = \frac{1}{J}\sum_{j=1}^{J} \bar{x}_j,$$

where $\bar{x}_j$ is mean value of chain j

$$\bar{x}_j = \frac{1}{L}\sum_{i=1}^{L} x_i^{(j)}.$$

The variance of the means of the chains

$$B = \frac{L}{J-1}\sum_{j=1}^{J}(\bar{x}_j - \bar{x}_*)^2,$$

and averaged variances of the individual chains across all chains is

$$W = \frac{1}{J}\sum_{j=1}^{J}\left(\frac{1}{L-1}\sum_{i=1}^{L}(x_i^{(j)} - \bar{x}_j)^2\right)$$

An estimate of the Gelman-Rubin statistic then results as

$$\hat{R} = \frac{\frac{L-1}{L}W + \frac{1}{L}B}{W}$$

Ideally we should get a value of 1, as the variance between chains should be the same as the variance within-chain. From a practical point of view values of $\hat{R} < 1.01$ are considered safe. According to the reported results, this condition also holds and the values of $\hat{R}$ are very close to one.

Monte Carlo Standard Error

When using MCMC methods, we introduce an additional layer of uncertainty because we are approximating the posterior with a finite number of samples. We can estimate the amount of error introduced using the Monte Carlo Standard Error (MCSE), which is based on the Markov Chain Central Limit Theorem. MCSE assumes that the samples are not truly independent of each other and is actually calculated from the ESS. While the ESS and $\hat{R}$ values are independent of the scale of the parameters, interpreting whether the MCSE is small enough requires expertise in the relevant field. If we want to report the value of an estimated parameter to the second decimal place, we need to ensure that the MCSE is below the second decimal place, otherwise we will mistakenly report a higher precision than we actually have. We should only check the MCSE when we are sure that the ESS is high enough and $\hat{R}$ is low enough. Otherwise, the MCSE is useless. Fortunately, our estimate gets a passing grade on this measure as well. Because its values are around 0.001.

6.2 Systematic Risk

Now, we come to the most interesting part: the analysis of the volatility variable. Figure 4 gives an overview of the exchange rate in Iran. The blue graph is the logarithm of volatility, and the red graph is the logarithm of the exchange rate. In the first episode, Iran faced increased volatility or systematic risk. In the second period, this problem was controlled, but with the implementation of the most severe round of sanctions, the risk reached its highest level. Not only did the systematic risk increase, but the volatility of the systematic risk also increased sharply. In the next two periods, the economy tried to recover, but it was unsuccessful in this matter, and the systematic risk was still at a high level. Extreme fluctuations in volatility are a new feature that has been added to the Iranian economy in the last three episodes. In the past, Iran faced rare currency jumps, but most of the time, the situation was calm. However, upon entering the third episode, the story changes completely. The fluctuations increase in both their mean and variance, which we also observed in the sigma parameter. Increased volatility of volatility shows us a high level of uncertainty. In other words, economic agents do not know exactly what risk they are facing, which means the same as Knightian uncertainty (see Knight [21]). This uncertainty disrupts almost all decisions because economic agents have very little information about the future. In such an environment, decisions by the government and other economic policymakers will not lead to an improvement in the situation. Therefore, the main priority of economic policymakers in the field of exchange rates should be to reduce this type of uncertainty.

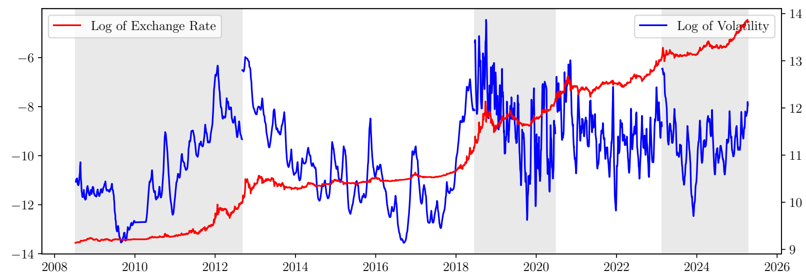

Figure 4: Overall Scheme of Systematic Risk

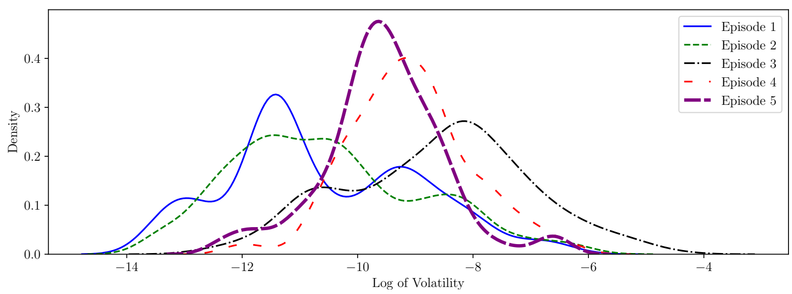

Figure 5: Distribution of Log of Volatility

Figure 5 looks at the problem from another perspective. In this figure, the distribution of logarithm volatility in five periods is displayed separately, and each color represents an episode. Episodes one and two have very similar distributions, which indicates the similarity of these two periods. Of course, we should not forget that in episode one, systematic risk got out of control and reached a high level, and in episode two, this problem was brought under control. In other words, these periods are the inverse of each other in terms of time. However, the peak of the distribution of the other three episodes is completely ahead, meaning that the average logarithm volatility is higher. The situation becomes more dangerous when we pay attention to the right tail of the distribution of episode three. This fat tail indicates a high probability of terrible events occurring in macroeconomics. Unlike other episodes, the probability of large shocks occurring in episode three is very high. Of course, after that, the economy recovered slightly, a significant part of which was due to the reduction of external shocks, but the average volatility is still very high. It should be noted that the scale of the x-axis is logarithmic, not volatility itself. In Figure 6, the x-axis is volatility itself, and it is clear how high the probability of large shocks occurring in the third episode was.

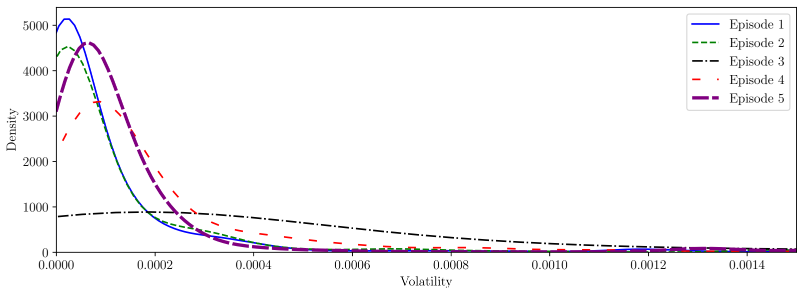

Figure 6: Distribution of Volatility

6.3 Volatility of Volatility

The Importance of Volatility of Volatility is an excuse to go into the exact calculation of the sigma effect. We start with the expectation of the logarithm of volatility with stationary measure

$$(\log V) = \frac{\alpha_v}{1-\beta_v}.$$

So, the expectation of V is equal to

$$[V] = e^{\frac{\alpha_v}{1-\beta_v}+\frac{\sigma_v^2}{2(1-\beta_v^2)}}. \tag{2}$$

We have $Y_t = \sqrt{V_{t-1}}\varepsilon_t^s$, that $\sqrt{V_{t-1}}$ and ε_t^s are independent. Therefore, the variance of Y_t is equal to

$$\begin{aligned}\operatorname{Var}(Y_t) &= \left[V_{t-1}\varepsilon_t^{s\,2}\right] - \left[\sqrt{V_{t-1}}\varepsilon_t^s\right] \\ &= [V_{t-1}]\left[\varepsilon_t^{s\,2}\right] - \left[\sqrt{V_{t-1}}\right][\varepsilon_t^s]\end{aligned}$$

According to $\varepsilon_t^s \sim \mathcal{N}(0,1)$, then we have

$$\operatorname{Var}(Y) = [V] \tag{3}$$

As a result, a decrease in sigma will lead to a decrease in the variance of daily returns. However, how much is this effect? We showed in the appendix 8 that the elasticity of the variance of daily returns of the exchange rate with respect to σ_v is equal to

$$\eta = \frac{\sigma_v^2}{1-\beta_v^2}$$

This means that under current conditions, assuming that other parameters remain constant, every 1% decrease in volatility will lead to approximately a 1.7% decrease in the variance of daily exchange rate returns, which is a significant number. Therefore, volatility can be an important variable for policymaking.

The important question that arises here is whether volatility is a policy variable at all? Volatility is a dynamic parameter of volatility, but unlike the other two parameters, it can be controlled. We propose that, similar to the idea of the Oil Stabilization Fund, the Central Bank should assume the role of a **risk stabilization fund**. The Central Banks role as a risk stabilization fund is to target a certain amount of volatility, given the macroeconomic situation and existing systemic risks, and to intervene in the market at times when the risk temporarily deviates from the targeted amount, so that the level of volatility returns to its long-term value. This idea could help reduce volatility if the central bank does not make errors in estimating the amount of volatility (or systematic risk).

7 Conclusion

The exchange rate in Iran, a pivotal oil country, holds a significant role in deciphering the macroeconomic landscape. Its volatility, a key indicator, is crucial for our analysis. Stochastic volatility models, a robust framework, aid in understanding the volatility of a time series. However, the crux lies in estimating volatility, a task for which several methods have been developed. We employed the Hamiltonian Monte Carlo method, known for its high estimation accuracy. In light of the political and economic shifts in Iran, we identified five distinct episodes for the exchange rate. To estimate the parameters of each episode, we utilized an autoregressive stochastic volatility model.

The results indicate that in less risky episodes, the persistence of volatility shocks is very high. However, in more risky periods, this parameter decreases, and then the average volatility increases. Another important result is that after the second round of sanctions, the economy has gone through three risky episodes. The important feature of exchange rate volatility in these three periods is that not only the mean but also the variance of volatility has increased. This means that we do not know exactly what volatility we are facing, which is a high level of uncertainty. The story of the exchange rate in Iran has many implications for policymakers. The main priority should be to reduce volatility, but if that is not possible, at least reduce volatility of volatility so that the pricing process is done correctly, emphasizing the urgency of the issue. Our estimates using the stochastic volatility model show that a 1% decrease in volatility will lead to a 1.7% decrease in the variance of daily exchange rate returns. We suggest that if the central bank is unable to control inflation due to the government's financial dominance, it should control the variance of daily exchange rate returns so that the economy faces fewer fluctuations. The central bank's tool for doing this is to target average volatility and attempt to reduce volatility through intervention in the foreign exchange market.

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8 appendices

Elasticity of Daily Return on σ_v

According to Equation 2, the derivative of the expectation of V with respect to σ_v is

$$\frac{[V]}{\sigma_v} = \frac{\sigma_v}{1-\beta_v^2} e^{\frac{\alpha_v}{1-\beta_v} + \frac{\sigma_v^2}{2(1-\beta_v^2)}}$$

Therefore, the elasticity of daily return on σ_v is equal to

$$\eta = \frac{[V]}{\sigma_v}\frac{\sigma_v}{[V]} = \frac{\sigma_v^2}{1-\beta_v^2}$$

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