Minimum Loss Design of $\overline{X}$ Control Chart for Correlated Data Under Weibull In-Control Times with Multiple Assignable Causes

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Abstract:

A proper method of monitoring a stochastic system is to use the control charts of statistical process control in which a drift in characteristics of output may be due to one or several assignable causes. In the establishment of $\overline{X}$ charts in statistical process control, an assumption is made that there is no correlation within the samples. However, in practice, there are many cases where the correlation does exist within the samples. It would be more appropriate to assume that each sample is a realization of a multivariate normal random vector. Using three different loss functions in the concept of quality control charts with economic and economic statistical design leads to better decisions in the industry. Although some research works have considered the economic design of control charts under single assignable cause and correlated data, the economic statistical design of $\overline{X}$ control chart for multiple assignable causes and correlated data under Weibull shock model with three different loss functions have not been presented yet. Based on the optimization of the average cost per unit of time and taking into account the different combination values of Weibull distribution parameters, optimal design values of

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sample size, sampling interval and control limit coefficient were derived and calculated. Then the cost models under non-uniform and uniform sampling scheme were compared. The results revealed that the model under multiple assignable causes with correlated samples with non-uniform sampling integrated with three different loss functions has a lower cost than the model with uniform sampling.
Keywords: Economic statistical design, $\overline{X}$ control chart, Multiple assignable causes, Weibull shock model, Correlated data, Taguchi loss function, Linear loss function, Exponential loss function..
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1. Introduction

The method of SPC firstly started from Shewhart's control charts. Control charts can be used for monitoring the production process and eliminating the effect of assignable causes. Various control chart techniques have been developed and widely applied in industries. In designing any control chart, three fundamental questions need to be answered about so-called design parameters. First, what should the sample size be? Second, how often should the samples be taken? Third, what should the control limits coefficient be? The optimal choice of chart parameters has a huge impact on the performance of a control chart.

In the existing literature, different methods have been developed for deciding the design parameters. The first method was the heuristic method in which some of the quality control Gurus suggested different values for the design parameters. Then, statistical methods were used to calculate the optimal values of the design parameters. Girshick and Rubin (1952) presented the concept of economic design for the first time and their study became a basis for subsequent research. Duncan (1956) in his paper adopted the economic design of $\overline{X}$ control charts under the exponential shock model. Although the economic consequences are considered in the economic design, this design is very poor in terms of statistical criteria (Woodall (1986)). To remove the weaknesses of the economic model, Saniga (1989) proposed a new model to combine the benefits of both pure statistical and economic designs while minimizing their weaknesses. This leads to propose the economic statistical design, where statistical constraints are incorporated into the economic design. Thus, it is one of the most versatile approaches in control chart designs, as it considers both cost and statistical performance of the control charts.

In Duncan's paper Duncan (1956), only single assignable cause made a shift in process mean. In industry, there are some situations when multiple assignable causes affect the model. Therefore, many researchers are interested in presenting a model in these situations (Gibra (1981);Chung (1991);Yang and *et al.* (2010)). Duncan (1971) extended his model from single assignable cause to multiple assignable causes where the assignable causes occurred independently. Based on Duncan's model, Yu and *et al.* (2010) presented an economic statistical design for control chart with multiple assignable causes and imposed constraints on Type I and Type II errors.

Economic and economic statistical design of control charts need to have a probability distribution for a process failure mechanism to put process costs in one model. Transition in the process from the state of control to out of control is called process failure mechanism (PFM) or shock model. A lot of distributions such as Exponential, Weibull, Generalized exponential, Burr 12, Gamma, Pareto and. . . ., are used as a failure mechanism (Duncan (1956);Banerjee and Rahim

(1988); Moghadam and *et al.* (2016);Heydari and *et al.* (2016);Pasha and *et al.* (2017);Al-Oraini and Rahim (2002);Kraleti and Kambagowni (2010).These distributions have applications in other fields such as the distribution of life. These distributions have fixed, decreasing and increasing failure rates.

Since using the distributions with increasing failure rate corresponds to reality in the industry, Banerjee and Rahim (1988) used the Weibull distribution instead of exponential distribution to generalize Duncan (1956) model under non-uniform sampling scheme. Based on the cost model of Banerjee and Rahim (1988) and Duncan (1971) model Chen and Yang (2002) presented the economic design of control charts under Weibull shock model with multiple assignable causes and variable sampling intervals.

The measurements within the samples in the above-mentioned models are totally assumed to be independently distributed in the design procedure of a control chart. Leavenworth and Grant (2000) stated that this assumption may not be defendable in some specific processes; for example, the collected measurements within a sample from the production process, which comprises multiple but similar characteristics in a single part. Other specific examples include several cavities on a single casting, multiple pins on an integrated circuit chip, or multiple contact pads on a single machine mount, which may be correlated. Neuhardt (1987) investigated the effects of correlation existing within a subgroup in a control chart. Yang and Hancock (1990) extended Neuhardt's work to determine the effect of correlated data on $\overline{X}$, R, S and S^2charts by Monte Carlo Simulation studies. Chou and *et al.* (2001) combined the Yang and Hancock (1990) model with the economic design approach to determine the parameters of average control charts under correlated samples. Liu and *et al.* (2002) employed Yang and Hancock's correlation model and the fixed-sampling-interval (FSI) policy to develop a minimum-loss design of $\overline{X}$ charts for correlated data. Chen and *et al.* (2007) combined Banerjee and Rahim (1988) cost model with Yang and Hancock (1990) correlation model to develop an economic statistical design model of $\overline{X}$ charts for processes with correlated data and the Gamma failure mechanism.

The use of control charts implies that quality loss is considered as the cost when the quality characteristics are outside the specification limits. All products falling within the control limits are considered as having the same quality regardless of the deviation of their quality characteristic from its target value. However, this is not the case when it comes to real life examples in which any deviation from the target value will incur a cost to the customers. Taguchi and *et al.* (1989) defined quality loss as "the loss to society caused by the product after it is shipped out". His quadratic loss function is well known and has been widely used in all fields. It is used to estimate the quality loss of a product when its quality characteristic de-

viates from its target value. Until now, a lot of economic and economic statistical design developed for control chart by the combination of classic models like Duncan and Lorenzen Vance model by Taguchi loss function (Safaei and *et al.* (2012); Al-Ghazi and *et al.* (2007); Yang (1998); Koo and Lin (1992)). In economic and economic statistical design, loss cost in control and out of control time is calculated with Taguchi loss function in many researches (Serel and *et al.* (2003); Elsayed and Chen (1994). Yu and Chen (2009) presented the economic statistical design of control chartwith Taguchi loss function under multiple assignable causes.
We consider linear, Taguchi (quadratic), and exponential loss functions which are commonly used in the literature (Elsayed and Chen (1994); Moskowitz and *et al.* (1994); Serel and *et al.* (2003). For a given deviation from target, the implied quality cost depends on the loss function used. Using numerical examples, we explore the impact of the form of the loss function on the chart parameters minimizing the overall cost.
The economic statistical design of $\overline{X}$ control chart under Weibull shock model for correlated data with multiple assignable causes and three different loss functions is not presented yet and this paper presents economic statistical design of $\overline{X}$ control chart under Weibull shock model with multiple assignable causes and three different loss functions for correlated data by using the concepts of the average time since the occurrence of an assignable cause until the chart alarms (AATS) and the expected number of false alarms(ANF). In this paper, by considering fixed sampling interval (uniform sampling scheme), we calculate the average cost of the cycle and compare our findings with the average cost in the case of non-uniform sampling. To calculate cost functions for uniform and non-uniform sampling schemes, this study has presented and proved the formulas of based on multiple assignable causes and in the case of uniform and non-uniform sampling schemes. To construct economic statistical design we used penalty approach and both the statistical properties and optimization of loss cost have been considered simultaneously.

The structure of this paper is as follows. In the second Section, some essential points are given about $\overline{X}$ Control Chart under Weibull In-Control Times. In section three some necessary points are given. In the fourth part, a cost model offered with multiple assignable causes with non-uniform and uniform sampling schemes. In this section integrated cost model with three different loss functions also were presented. Section 5 includes the economic statistical design. Section 6 includes real industrial example. The determination of input parameters and optimizing cost model based on this input parameters by considering economic and economic statistical designs are also presented in this section. The comparison between cost model under multiple assignable causes with uniform and non-uniform schemes are also presented in Section six. Finally, a brief summary appears in the last section.

2. $\overline{X}$ Control Chart under Weibull In-Control Times

When designing a control chart, one usually assumes the measurements within the sample are independently distributed. However, this assumption may not be tenable for some specic processes. Yang and Hancock (1990) assume that each subgroup (samples of size n from X in sampling intervals) is a realization of the random vector,$X = X_1, X_2, X_3, ..., X_n$, which has the multivariate normal distribution $N(\mu, V)$, where μ is the vector of mean values and $V = V_{ij}, i, j = 1, 2, ..., n$, is the covariance matrix. In addition, , is the correlation matrix and σ is the process standard deviation.

Lemma 2.1. *In the case of correlated samples assumptions, the sample mean $\overline{X}$ can be shown to be normally distributed with mean and variance as follows:*

$$E(\overline{X}) = \mu \tag{2.1}$$

$$V(\overline{X}) = \tfrac{\sigma^2}{n}[1 + (n-1)\rho] \tag{2.2}$$

where

$$\rho = \frac{\sum_{i \neq j} r_{ij}}{n(n-1)} \tag{2.3}$$

the proof is given bellow

Proof. Recall that the $X \sim N(\mu, V)$, with $V = \sigma^2 R$ is the process variance and R is the correlation matrix.

$$\begin{aligned} Var(\overline{X}) = & \tfrac{1}{n^2}[\textstyle\sum_{i=1}^{n} Var(X_i) + \sum_{i \neq j} Cov(X_i, X_j)] = \tfrac{1}{n^2}[n\sigma^2 + \sum_{i \neq j} V_{ij}] \\ & = \tfrac{1}{n^2}[n\sigma^2 + \sigma^2\{\textstyle\sum_{i \neq j} r_{ij}\}] \end{aligned}$$

Let $\rho = \frac{\sum_{i \neq j} r_{ij}}{n(n-1)}$ then

$$Var(\overline{X}) = \frac{1}{n^2}[n\sigma^2 + \sigma^2 n(n-1)\rho] Var(\overline{X}) = \frac{\sigma^2}{n}[1 + (n-1)\rho]$$

□

In this paper we have S assignable causes affected the process. It is assumed that the time of being in control until i^{th} assignable cause occurs follows a Weibull distribution with bellow probability density function and increasing hazard rate:

$$f_i(t) = \lambda_i k t^{k-1} exp(-\lambda_i t^k).(\quad t > 0, k \geq 1, \lambda_i > 0), i = 1, 2, ..., s. \tag{2.4}$$

$$r_i(t) = \lambda_i k t^{k-1} \tag{2.5}$$

where k is shape parameter and λ_i is scale parameter. Note that when $k = 1, f_i(t)$ becomes exponential distribution with $r_i(t) = \lambda_i$. The process is monitored by taking samples of size n from X at time intervals h_1 $h_1 + h_2$ $h_1 + h_2 + h_3$ and so on. In this case is j^{th} sampling interval and we have $h_1 \geq h_2 \geq h_3, ...$
Where

$$W_j = \Sigma_{i=1}^{j} h_i \tag{2.6}$$

Lemma 2.2. *Let h_j is the j^{th} sampling interval. Then to keep the probability of shift from a control state fixed for all intervals, we have*

$$h_j = [j^{\frac{1}{k}} - (j-1)^{\frac{1}{k}}] h_1 \tag{2.7}$$

Proof.

$$\int_{\omega_j}^{\omega_{j+1}} r_i(t)\,\mathrm{d}t = \int_{\omega_0}^{h_1} r_i(t)\,\mathrm{d}t, j = 1, 2, ...$$
$$\int_{\omega_j}^{\omega_{j+1}} \lambda_i k t^{k-1}(t)\,\mathrm{d}t = \int_{\omega_0}^{h_1} \lambda_i k t^{k-1}(t)\,\mathrm{d}t = \omega_{j+1}^k - \omega_j^k = h_1^k$$
$$If j = 1 : \omega_2^k = \omega_1^k + h_1^k \Rightarrow \omega_2 = 2^{\frac{1}{k}} h_1$$
$$If j = 2 : \omega_3^k = \omega_2^k + h_1^k \Rightarrow \omega_3 = 3^{\frac{1}{k}} h_1$$
$$\vdots$$
$$\Rightarrow \omega_j = j^{\frac{1}{k}} h_1$$

But

$$h_j = \omega_j - \omega_{j-1}$$
$$h_j = [j^{\frac{1}{k}} - (j-1)^{\frac{1}{k}}] h_1, \qquad j = 1, 2, ...$$

□

Lemma 2.3. *The probability density function of occurrence of multiple assignable causes follows Weibull distribution.*

$$f_0(t) = \lambda_0 k t^{k-1} exp(-\lambda_0 t^k).(\quad t > 0, k \geq 1, \lambda_0 > 0) \tag{2.8}$$

where $\lambda_0 = \sum \lambda_i, i = 1, 2, ..., s.$

Proof. We assumed that S assignable causes affected the process and the occurrence time of any assignable cause follows Weibull distribution. It is also assumed that after the occurrence of the i_{th} assignable cause, until the discovery of the i_{th} assignable cause, the process will not disturb by any other assignable causes. Thus, if the time until occurrence of assignable causes noted by $T_1^{'}, T_2^{'}, ..., T_S^{'}$ then the probability of being in control at time is:

$$P(T^{'} > t) \quad = P(min(T_1^{'}, T_2^{'}, ..., T_S^{'}) > t) \quad = \exp^{-\lambda_0 t^k}$$

where $\lambda_0 = \sum \lambda_i, i = 1, 2, ..., s.$ □

Lemma 2.4. *The probability of the occurrence of the ith assignable cause before the other assignable causes is*

$$\frac{\lambda_i}{\lambda_0} \tag{2.9}$$

Proof. if C_i is the event of the occurrence of i^{th} assignable cause, then:

$$C_i = [A_i = U] = [A_i < min(A_1, A_2, ..., A_{i-1}, A_{i+1}, ..., A_s)]$$

then if we have $R = min(A_1, A_2, ..., A_{i-1}, A_{i+1}, ..., A_s)$

$$F_{A_i}(r) = 1 - exp(-\lambda_i r^k)$$
$$f_R(r) = k(\lambda_0 - \lambda_i) r^{k-1} exp(-(\lambda_0 - \lambda_i) r^k)$$

We have:

$$\begin{aligned} P(C_i) = P[A_i < R] &= \int_0^\infty P(A_i < R \mid R = r) f_R(r)\,\mathrm{d}r \\ &= \int_0^\infty P(A_i < r) f_R(r)\,\mathrm{d}r \\ &= \int_0^\infty F_{A_i}(r) f_R(r)\,\mathrm{d}r \\ &= \int_0^\infty k(\lambda_0 - \lambda_i) r^{k-1} exp(-(\lambda_0 - \lambda_i) r^k)\,\mathrm{d}r \\ &= 1 - \int_0^\infty k(\lambda_0 - \lambda_i) r^{k-1} exp(-\lambda_0 r^k)\,\mathrm{d}r \\ &= \frac{\lambda_i}{\lambda_0} \end{aligned}$$

□

3. Performance indicators

There are several statistical measures to assess the performance of $\overline{X}$ control charts for correlated data under Weibull shock model, such as:
1. Adjusted Average Time to Signal ($AATS$) is defined as the average time from when the process shifts until the chart gives an out-of-control signal.
2.The ANF is defined as the expected number of false alarms.
3.The ANS_0 is defined as the expected number of samples in the in control period.

We need to define the following terms to calculate the Performance indicators.
1. p_{ij} is the conditional probability that i^{th} assignable cause will occur during j^{th} sampling interval , given that i^{th} assignable cause not occur at time ω_{j-1}.

$$p_{ij} = \frac{\int_{\omega_{j-1}}^{\omega_j} f_i(t)\,\mathrm{d}t}{\int_{\omega_{j-1}}^{\infty} f_i(t)\,\mathrm{d}t} \tag{3.10}$$

$$= \frac{exp(-\lambda_i \omega_{j-1}^k) - exp(-\lambda_i \omega_j^k)}{exp(-\lambda_i \omega_{j-1}^k)} \tag{3.11}$$

$$= 1 - exp(-\lambda_i(jh_1^k)$$

let $p_{ij} = p_i$, for $(i = 1, 2, ..., S), (j = 1, 2, ...)$.
According to the above formula, we consider p_{0j} as the conditional probability that multiple assignable causes will occur during j^{th} sampling interval given that multiple assignable causes do not occur at time ω_{j-1}. We obtain

$$p_{0j} = 1 - e^{-\lambda_0 h_1^k}, \qquad j = 1, 2, ... \tag{3.12}$$

Here we assumed $p_{0j} = p_0$.
2. We consider q_{ij} as the unconditional probability that i^{th} assignable cause will occur during j^{th} sampling interval and the process is going to out of control.

$$q_{ij} = \int_{\omega_{j-1}}^{\omega_j} f_i(t)\,\mathrm{d}t = e^{-\lambda_i \omega_{j-1}^k} - e^{-\lambda_i \omega_j^k} = (1 - p_i)^{j-1} p_i \tag{3.13}$$

3. Suppose that τ_{ij} be the expected time of the in control period within sampling interval h_j , given that i^{th} assignable cause has occurred during this period.

$$\tau_{ij} = E(T - \omega_{j-1} \mid \omega_{j-1} < T < \omega_j) = \frac{\int_{\omega_{j-1}}^{\omega_j} (t - \omega_{j-1}) f_i(t)\,\mathrm{d}t}{q_{ij}} \tag{3.14}$$

Lemma 3.1. *The expected (the time that process be under control) during any one sampling interval is as follows:*

$$\tau_i = (\frac{1}{\lambda_i})^{\frac{1}{k}} \Gamma(1 + \frac{1}{k}) - h_1 p_i (1 - p_i) A(1 - p_i) \tag{3.15}$$

where for $|x| < 1$

$$A(X) = \sum_{j=0}^{\infty} (j+1)^{\frac{1}{k}} X^j \tag{3.16}$$

Proof.

$$\begin{aligned}\tau_i &= \sum_{j=1}^{\infty} \tau_{ij} q_{ij} = \sum_{j=1}^{\infty} \int_{\omega_{j-1}}^{\omega_j} t f_i(t)\,\mathrm{d}t - \sum_{j=1}^{\infty} q_{ij}\omega_{j-1} \\ &= (\frac{1}{\lambda_i})^{\frac{1}{k}} \Gamma(1+\frac{1}{k}) - h_1 p_i (1-p_i) A(1-p_i)\end{aligned}$$

□

Lemma 3.2. *Let AATS be the average time from when the process shifts until the chart gives an out-of-control signal. It is equal to:*

$$AATS = \Sigma(\frac{\lambda_i}{\lambda_0}) AATS_i \tag{3.17}$$

where $AATS_i$ *be the average time between occurrence shifts in process mean owing to the* i^{th} *assignable cause and receiving right alarm from control chart:*

$$AATS_i = h_1 p_i A(1-p_i) + \frac{\beta_i h_1 p_i [p_i A(1-p_i) - (1-\beta_i) A(\beta_i)]}{1-p_i-\beta_i} - (\frac{1}{\lambda_i})^{\frac{1}{k}} \Gamma(1+\frac{1}{k}) \tag{3.18}$$

Proof.

$$\begin{aligned}AATS_i &= \sum_{j=1}^{\infty} [\sum_{k=1}^{\infty} q_{ij} [\omega_{k+j-1} - \omega_{j-1}] \beta_i^{k-1} (1-\beta_i)] - \tau_i \\ &= \sum_{j=1}^{\infty} [\sum_{k=1}^{\infty} (1-p_i)^{j-1} p_i [\omega_{k+j-1} - \omega_{j-1}] \beta_i^{k-1} (1-\beta_i)] - \tau_i \\ &= (1-\beta_i) p_i \sum_{j=1}^{\infty} [\sum_{k=1}^{\infty} [\underbrace{(1-p_i)^{j-1} \omega_{k+j-1} \beta_i^{k-1}} - \underbrace{(1-p_i)^{j-1} \omega_{j-1} \beta_i^{k-1}}]] - \tau_i\end{aligned}$$

For the first part, we have:

$$\begin{aligned}I &= \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} (1-p_i)^{j-1} \omega_{k+j-1} \beta_i^{k-1} = \sum_{l=1}^{\infty} \omega_l \beta_i^l \sum_{j=1}^{l} (1-p_i)^{-1} (\frac{1-p_i}{\beta_i})^j \\ &= (\frac{1}{1-p_i}) \sum_{l=1}^{\infty} \omega_l \beta_i^l (\frac{1-p_i}{\beta_i - 1 + p_i}) - (\frac{1}{1-p_i}) \sum_{l=1}^{\infty} \omega_l (\frac{(1-p_i)^{l+1}}{\beta_i - 1 + p_i}) \\ &= (\frac{h_1}{p_i + \beta_i - 1}) (\beta_i A(\beta_i) - (1-p_i) A(1-p_i))\end{aligned}$$

For the second part, we have:

$$II = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}(1-p_i)^{j-1}\omega_{j-1}\beta_i^{k-1} = \sum_{j=1}^{\infty}(1-p_i)^{j-1}\omega_{j-1}\sum_{k=1}^{\infty}\beta_i^{k-1} = \frac{h_1(1-p_i)}{1-\beta_i}A(1-p_i)$$

By substituting and simplifying, final formula is obtained. □

In $\overline{X}$ control chart with correlated data the probability of Type II error is calculated as follows:

$$\beta_i = \int_0^{L-\delta_i\sqrt{n}} \frac{1}{\sqrt{2\pi}}exp(-\frac{1}{2}x^2)\mathrm{d}x - \int_0^{-L-\delta_i\sqrt{n}} \frac{1}{\sqrt{2\pi}}exp(-\frac{1}{2}x^2)\mathrm{d}x \tag{3.19}$$

The concept of $AATS_i$ is presented well in Figure 1.

Lemma 3.3. *Let is ANS_0 defined as the expected number of samples in the in control period. It is equal to:*

$$ANS_0 = \frac{1-p_0}{p_0} \tag{3.20}$$

Proof. If is the event of the occurrence of single assignable cause, then the expected number of samples in the in control period calculated as follows:

$$\begin{aligned} E(Number\ of\ samples\ are\ taken\ before\ shift) &= \sum_{j=0}^{\infty} jP(A \in (jh,(j+1)h) \\ &= \sum_{j=0}^{\infty} j(e^{-\lambda_0 j h_1^k} - e^{-\lambda_0 (j+1) h_1^k}) = \frac{e^{-\lambda_0 h_1^k}}{1-e^{-\lambda_0 h_1^k}} \end{aligned}$$

We obtain before: $p_0 = 1 - e^{-\lambda_0 h_1^k}$. □

The expected number of false alarms generated during a cycle is times the expected number of samples taken before the shift, or

$$ANF = \alpha ANS_0 \tag{3.21}$$

In $\overline{X}$ control chart with correlated data the probability of Type I error is calculated as follows:

$$\alpha = 2\int_L^{\infty} \frac{1}{\sqrt{2\pi}}exp(-\frac{1}{2}x^2)\mathrm{d}x \tag{3.22}$$

4. Development of cost model

4.1 Model Assumptions

To create a cost model we should consider the following assumptions:
1. The output of the process has a normal distribution with constant mean and variance.
2. When the process is under the control, $\mu = \mu_0$.
3. It is assumed that assignable causes occur based on the Weibull distribution. By assuming that the process begins with the state of control, the time which process be under control has a Weibull distribution.
4. Assignable causes occur independently.
5. Multiple assignable causes produce "step changes" in the process mean from $\mu = \mu_0$ to a $\mu = \mu_0 + \delta_i\sigma$.
6. In this article, the shift occurred in the process mean is noted by δ_i. Three distributions, uniform, negative exponential and half-normal are considered as a prior for δ_i. Considering these distributions as the prior would cover all values of δ_i in a real industry.
7. The process is not self-correcting. That is, once a transition to an out-of-control state has occurred, the process can be returned to the in-control condition only by management intervention upon appropriate corrective actions.
8. The quality cycle starts with the in-control state and continues until the process is repaired after an out-of-control signal. It is assumed that quality cycle follows a Renewal Reward Process.
9. During the search for an assignable cause, the process is shut down.
10. Each subgroup (samples of size n from X in each sampling interval) is a realization of the random vector, $X = X_1, X_2, ..., X_n$, which has the multivariate normal distribution $N(\mu, V)$, where μ is the vector of mean values and $V = V_{ij}, i, j = 1, 2, ..., n$, is the covariance matrix.

To facilitate exposition, all notations used throughout this paper are summarized bellow:

Z_0: Average time to search for false alarm.
Z_1: Average time to discover assignable cause once it is detected by control chart.
Z_{2i}: Average time to repair i^{th} assignable cause after it has been discovered.
D_0: Average cost per unit of time while the process in control.
$D1i$: Average cost per unit of time while the process is out of control owing to the occurrence of the i^{th} assignable causs
J_0: In control cost obtained by considering loss function.
J_{1i}: Out of control cost obtained by considering loss function.

Y: The average cost per false alarm when the process is under control.
W_i: Cost to locate and repair ith assignable cause.
a: Fixed sample cost.
b: Unit sample cost
P: Production rate.
σ: Standard deviation of the process.
λ_i: Weibull Scale parameter
k: Weibull Shape parameter
n: Sample size
h: Sampling interval
L: Control limits coefficient
ρ: Average correlation factor within samples

4.2 Cost function in the case of non-uniform sampling

In practice, each process starts from in control state. Then because of occurrence of one assignable cause, it goes to out of control state. It is clear that after repairing and fixing the assignable cause, the process returns to the initial state. This cycle is called quality cycle and its model follows the form of a Renewal Reward Process where the average cost per unit time for the cycle $E(A)$ is calculated by the average cost per cycle $E(C)$ divided by the average time per cycle $E(T)$. In economic design, the purpose is optimizing without any constraint and finding optimal values for sampling interval, sample size, and control limits coefficient. The average time that the process is in control is:

$$(\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) + Z_0 ANF \tag{4.23}$$

The average time of cycle is:

$$E(T) = (\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) + Z_0 ANF + AATS + Z_1 + \Sigma(\frac{\lambda_i}{\lambda_0})Z_{2i} \tag{4.24}$$

For better understanding of $E(T)$ one can see Figure 1.
The average cost of cycle is:

$$E(C) = D_0(\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) + YANF + \sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}D_{1i}AATS_i + \sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}w_i \tag{4.25}$$
$$+ (a+bn)\sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}(\frac{1}{p_0} + \frac{\beta_i}{1-\beta_i})$$

For better understanding of $E(C)$ one can see Figure 1.

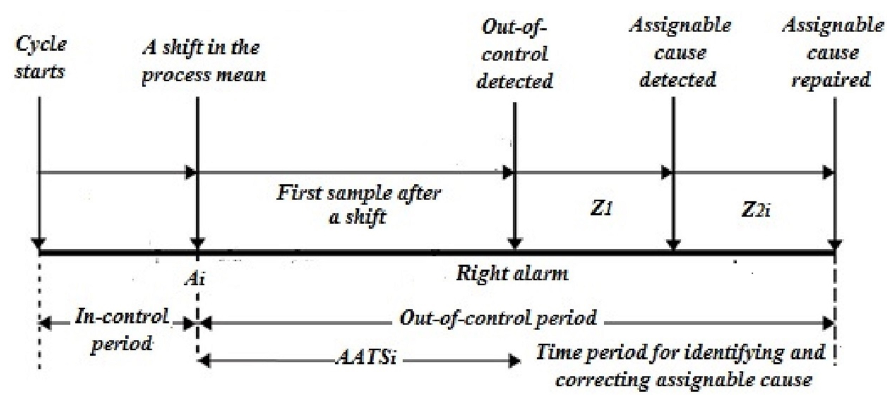

Figure 1: Quality Cycle

4.3 Cost function in the case of uniform sampling

To evaluate the relative benefits of non-uniform sampling plan in comparison with uniform sampling plan under multiple assignable causes cost model, and by considering fixed sampling interval, we calculate average time and the average cost for the cycle and analyze them. If is a fixed sampling interval, then we can obtain $E(T)$ as follows:

$$E(T) = (\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) + Z_0 ANF + AATS + Z_1 + \Sigma(\frac{\lambda_i}{\lambda_0})Z_{2i} \tag{4.26}$$

where

$$AATS = \sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} AATS_i \qquad AATS_i = \frac{h}{1-\beta_i} - \tau_i$$

$$\tau_i = (\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) - hQ_i \qquad Q_i = \sum_{j=1}^{\infty} e^{\lambda_i (jh)^k}$$

$$ANF = \alpha Q, Q = \sum_{j=1}^{\infty} e^{\lambda_0 (jh)^k}$$

We also obtain $E(C)$ as fallows:

$$E(C) = D_0(\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) + YANF + \sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0} D_{1i} AATS_i + \sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0} w_i \tag{4.27}$$

$$+ (a+bn)\sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}(\frac{1}{1-\beta_i}) + (a+bn)Q$$

4.4 Improvement of Cost function by using loss function

In the traditional formulation of economic design models, the costs due to non-conformities when the process is in-control (D_0) and out-of-control (D_{1i}) have been treated as constants. In recent years, influenced in part by the popularity of Taguchi methods in product design, the quality loss function concept has been incorporated into various statistical decision models where the cost due to poor quality needs to be estimated. In the traditional approach, the upper and lower specification limits are used to classify the quality of the process output as either acceptable or non-acceptable, and products falling outside the specification limits are considered to result in quality costs. In the loss function approach, the probability distribution describing the observations for the quality characteristic is explicitly taken into account in computing the costs resulting from variation of the quality characteristic around its target. It is considered that cost of poor quality is incurred whenever the quality characteristic is not on its target; hence, products that are not produced on-target incur cost even though they may conform to specification limits. Several researchers have applied the loss function approach in the economic design of $\overline{X}$ control charts (Elsayed and Chen (1994); Moskowitz and *et al.* (1994). In this paper, we propose the economic design of charts based on linear, quadratic, and exponential loss functions.
We note that the appropriate type of the loss function to be used depends on the particular industrial application. It is also possible that the relevant loss function may be different for negative and positive deviations from the target. Cain and Janssen (1997) discuss a problem arising in the production of construction panels made of glued and pressed wood chips. The moisture level can be reduced by drying the panels longer in the gas dryers. The longer drying time requires more fuel to be consumed; hence, there is a linear increase in cost as the moisture content decreases. On the other hand, higher moisture increases the press time which increases the total plant operating cost. Thus, the resulting cost function is partly linear and partly quadratic. The cost increases quadratic when the moisture content is higher than planned; however, the cost increases linearly when the moisture content is lower than planned. Although, as in this example, the form of the loss function for the quality characteristic may be region-dependent, in this paper we will focus on the simpler and more common case where the loss function is single-type and symmetric around the target value. But, if needed, these more generalized (mixed type and asymmetric) loss functions can be easily incorporated into our model. We remark that different types of loss functions can also be regarded as reflections of varying risk preferences of the users of control charts. In this case, the quality loss function is related to the user's utility function.

4.4.1 Linear loss function

We first consider the linear loss function in which quality loss is a linear function of the deviation of the quality characteristic from its target. Let T be the target value for the quality characteristic monitored; we allow the possibility that T can be different from μ_0. Let the probability density function (pdf) of the quality characteristic X be $f(x)$. The quality loss is zero only when the quality characteristic X equals the target T, and the loss increases as the deviation from the target increases. If the loss function $L(X)$ is asymmetric around the target, two different loss coefficients C_1 and C_2 should be estimated such that the loss is calculated as

$$L(X) = C_1(T - X) if X \leq T, \tag{4.28}$$

$$C_2(X - T) if X > T. \tag{4.29}$$

If we consider the symmetric loss functions and we assume that the loss coefficient used for estimating the cost due to nonconformities $C = C_1 = C_2$, then the Linear loss function is obtained as follows:

$$L(X) = C|T - X| \tag{4.30}$$

Lemma 4.1. *If Z is a Standard Normal random variable then:*

$$E[|Z - a|] = 2[\phi(a) + a\Phi(a)] - a \tag{4.31}$$

the proof is given bellow:

$$\begin{aligned} E[|Z-a|] &= \int_{-\infty}^{a} (a-z)\phi(z)dz + \int_{a}^{\infty} (z-a)\phi(z)dz \\ &= 2a\Phi(a) - a - \int_{-\infty}^{a} z\frac{1}{\sqrt{2\pi}}e^{(}\frac{-z^2}{2})dz + \int_{a}^{\infty} z\frac{1}{\sqrt{2\pi}}e^{(}\frac{-z^2}{2})dz \\ &= 2[\phi(a) + a\Phi(a)] - a \end{aligned}$$

When the process is in the control state, quality characteristic has the Normal distribution with the mean parameter μ_0 and the variance parameter σ^2, therefore according to above lemma, and considering $a_0 + \frac{(T-\mu 0}{\sigma}$, $f(x) = (2\pi\sigma^2)^{-0.5}e^{\frac{(x-\mu_0^2}{2\sigma^2}}$, and $\Phi(.)$ is the cumulative probability distribution function (cdf) for a standard normal variable and $\phi(.)$ is the standard normal pdf. The expected quality cost per unit when the process is in control, J_0,is:

$$J_0 = 2C[\sigma\phi(a_0) - (\mu_0 - T)\Phi(a_0)] - C(\mu_0 - T). \tag{4.32}$$

Let the out-of-control process mean be $\mu_{1i} = \mu_0 + \delta_i\sigma$. Defining $a_{1i} = \frac{T-\mu_i}{\sigma}$, the expected quality cost per unit when the process is out of control, J_{1i}, according to above formula is:

$$J_{1i} = 2C[\sigma\phi(a_{1i}) - (\mu_{1i} - T)\Phi(a_{1i})] + C(\mu_{1i} - T) \tag{4.33}$$

If P units are produced per hour, we can compute D_0 and D_{1i} . Note that the shift in mean $\delta_i\sigma$ explicitly enters the cost function through the term J_{1i} when a loss function is used for computing the quality costs.

4.4.2 Quadratic loss function

The most common loss function used in practice is the symmetric quadratic loss function advocated by Taguchi. The quadratic loss function penalizes the deviations from the target more severely than the linear loss function. Kim and Liao (1994) suggest the liquid products in containers such as juice, soda, and medicine as potential applications of the symmetric quadratic loss function. Product delivery time promised to customers is an example of an asymmetric quadratic loss function. The actual delivery occurring earlier than the promised time incurs a small loss, which is considerably less than the loss associated with a late delivery resulting in customer dissatisfaction. Another example for the asymmetric loss is the contents of a manufactured drug. The low amount of a particular ingredient may make the drug ineffective, but the high level of the same ingredient may have a serious negative effect on users, implying that positive deviation from the target incurs a higher loss than the same amount of deviation below the target. Taguchi loss function is presented below:

$$L(X) = C(X - T)^2 \tag{4.34}$$

If X is a random variable with the mean parameter μ and the variance parameter σ^2 then we have:

$$E(X - a)^2 = Var(X) + (E(X) - a)^2 \tag{4.35}$$

When the process is in the control state, quality characteristic has the Normal distribution with the mean parameter μ and the variance parameter σ_2, therefore according above equation, we calculate J_0 as

$$J_0 = C[\sigma^2 + (\mu_0 - T)^2] \tag{4.36}$$

The expected cost per unit under quadratic loss function when the process is out of control is

$$J_{1i} = C[\sigma^2 + (\mu_0 - T)^2 + \delta_i^2\sigma^2 - 2\delta_i\sigma(\mu 0 - T)] \tag{4.37}$$

As in the case of linear loss function, the optimal design under quadratic loss function can be found by first finding D_0, D_1 using above formulas, and then substituting them in cost function.

4.4.3 Exponential loss function

Finally we consider the exponential loss function which corresponds to the case of constant risk aversion if we assume that the utility of the decision maker is measured by the negative of the quality loss Moskowitz and *et al.* (1994). The linear loss function is suited to a risk-neutral decision maker whereas the quadratic and exponential loss functions allow incorporation of risk aversion explicitly into the model. The choice of a quadratic loss function implies that the decision maker becomes less risk averse as the deviation of the quality characteristic from the target increases. The exponential loss function implies that the utility of the decision maker decreases exponentially as deviation from the target increases. The Exponential loss function is:

$$L(X) = E[C(e^{r|X-T|}) - 1] \tag{4.38}$$

Lemma 4.2. *If Z is a Standard Normal random variable then:*

$$E[|Z-a|] = 2[\phi(a) + a\Phi(a)] - a \tag{4.39}$$

the proof is given bellow:

$$\begin{aligned} E(e^{r|X-a|}) &= \int_{-\infty}^{a} e^{r(a-z)}\phi(z)dz + \int_{a}^{\infty} e^{r(z-a)}\phi(z)dz \qquad (4.40) \\ &= e^{ra}\int_{-\infty}^{a} e^{-rz}\frac{1}{\sqrt{2\pi}}exp(\frac{-z^2}{2})dz + e^{-ra}\int_{a}^{\infty} e^{rz}\frac{1}{\sqrt{2\pi}}exp(\frac{-z^2}{2})dz \\ &= exp(\frac{r^2}{2}[e^{ra}\phi(a+r) + e^{-ra}(1-\phi(a-r))] \end{aligned}$$

When the process is in the control state, quality characteristic has the Normal distribution with the mean parameter μ and the variance parameter σ_2, therefore according above lema, we calculate J_0 as

$$\begin{aligned} J_0 &= E[C(e^{r|X-T|}) - 1] = CE[(e^{r|\sigma z+\mu_0 - C|}) - 1] \\ &= C[exp(r\sigma b_0 + \frac{(r\sigma)^2}{2}) + \phi(b_0 + r\sigma) + exp(-r\sigma b_0 + \frac{(r\sigma)^2}{2})(1 - \phi(b_0 - r\sigma)) - 1 \end{aligned} \tag{4.41}$$

where $b_0 = \frac{T-\mu_0}{\sigma}$ and

$$\begin{aligned} J_0 = Cexp(\frac{(r\sigma^2)}{2} * [&exp(r(T-\mu_0))\Phi(b_0 + r\sigma) + exp(r(\mu_0 - T)) \\ &-exp(r(\mu_0 - T))\Phi(b_0 + r\sigma)] - C \end{aligned} \tag{4.42}$$

when the process is out of the control, quality characteristic has Normal distribution with the mean parameter $\mu_{1i} = \mu_0 + \delta_i\sigma$ and the variance parameter σ^2 .

According to the above lemma we have:

$$
\begin{aligned}
J_{1i} \qquad &= Cexp(\frac{(r\sigma^2}{2}) * [exp(r(T-\mu_{1i}))\phi((b_{1i}+r\sigma)) \\
&+exp(r(\mu_{1i}-T)) - exp(r(\mu_{1i})-T))\phi((b_{1i})+r\sigma] - C
\end{aligned} \tag{4.43}
$$

where $b_{1i} = \frac{T-\mu_{1i}}{\sigma}$.

5. Economic-Statistical design

In statistical design and economic design of control charts, optimal performance of design parameters obtained in terms of statistical and economic criteria, but in economic statistical design, statistical and economic criteria considered jointly. In this paper economic statistical design is derived based on minimizing average cost per time and by considering maximum values for the adjusted average time to signal ($AATS$) and average numbers of false alarm in the quality cycle (ANF). If we note the average cost of the cycle per time by $E(A)$ and the set of economic design parameters of $\overline{X}$ control charts by F, we can show the economic statistical design of $\overline{X}$ control charts as follows:

$$F(n, h_1, L) = Min\ E(A) \tag{5.44}$$

$$\begin{cases} AATS \leq AATS_u \\ ANF \leq ANF_u \end{cases}$$

Where $AATS_u$ and ANF_u are the corresponding bounds of values of $AATS$

and ANF. It should be noted that according to the values obtained in economic design, the upper limit of $AATS$ was considered 1 and the upper limit of ANF was considered 0.5.

6. Real Example and solution procedure

Here, an example is presented to illustrate the solution procedure of the economic statistical design of the charts for correlated data Chou and *et al.* (2001). A plant, located in central Taiwan, produces grape juice, which is contained in glass bottles. The target quantity of grape juice' is 200 cm3 for each bottle. In the production process, the grape juice is inserted into twelve bottles at a time, and the twelve bottles of juice will be packed in a box later. Before the twelve bottles of grape juice are packed, the inspector samples the first four bottles to check whether the quantity of grape juice for each bottle is $200cm^3$ and $\overline{X}$ chart is applied to monitor the process of insertion. The subgroups, that is, the first four bottles from the recent 100 successive boxes, are viewed as a random sample from

Table 1: The D_0 and D_1 values based on three different loss functions

Linear Loss		Quadratic Loss		Exponential Loss	
D_0	24.80	D_0	14.50	D_0	33.15
D_{11}	19.36	D_{11}	23.13	D_{11}	124.92
D_{12}	17.24	D_{12}	26.50	D_{12}	116.30
D_{13}	26.67	D_{13}	30.13	D_{13}	106.88
D_{14}	38.15	D_{14}	34.00	D_{14}	96.60
D_{15}	50.66	D_{15}	38.13	D_{15}	85.57
D_{16}	62.75	D_{16}	42.50	D_{16}	74.12
D_{17}	72.89	D_{17}	47.13	D_{17}	62.73
D_{18}	79.79	D_{18}	52.00	D_{18}	52.02
D_{19}	82.76	D_{19}	57.13	D_{19}	42.57
D_{110}	81.90	D_{110}	62.50	D_{110}	34.86

a multivariate distribution. Moreover, the vector of mean values is , the average correlation factor is estimated to be 0.1.

Some of the fixed parameters including cost parameters (Y, a, b) and time parameters (Z_0, Z_1) have been determined based on past experience. We also have ρ here as a fixed parameter. In the model W_i is non-fixed cost parameters and Z_{2i} is non-fixed time parameter. Weibull distribution parameters (λ_i, k), shift parameter δ_i and design parameters (n, h_1, L) are also presented in the model. In the numerical example, we assume: $Y = 1500, a = 25, b = 25, Z_0 = 1.35, Z_1 = 1.35, \rho = 0.1, P = 50$ The above parameters are not affected by the occurrence of different assignable causes and the shift created in the mean process. In this paper, we need some assumptions to estimate other parameters.

a) It is assumed X is quality characteristic, and assume that when the process is in-control:

$$X \sim N(\mu_0 = 1.5, \sigma_0 = 0.2), T = 2, r = C = 1 \tag{6.45}$$

D_0, D_{1i} is computed based on three kinds of loss function.

c) Suppose that $\delta_i = 1.75$ is a base case. In this case, assume that $Z_{2i} = 2.5, w_i = 750$. Banerjee Rahim (1988) single assignable cause model is compared with our multiple assignable causes model. Base case parameters are also considered for single assignable cause model $(w = 750, Z_2 = 2.5, \lambda = 0.02, \delta = 1.75, \rho = 0.1)$.

d) We assume that process is disturbed by ten assignable causes which produce ten shifts amount in the process mean vector that is from 0.75 to 3 in steps of 0.25.

In this article, we noted the prior distribution for δ_i by PD_i. As mentioned earlier three distribution uniform, negative-exponential and half-normal are considered as a prior for δ_i. Based case are considered for $\delta_i = 1.75$. PD_5 is the notation for base case prior distribution. The amount of Weibull scale parameter are calculated

Table 2: Model input parameters

		PD_i			Z_{2i}			w_i			λ_i		
A_i	δ_i	NE	Un	HN	NE	Un	HN	NE	Un	HN	NE	Un	HN
1	0.75	0.344	0.100	0.372	4.122	2.5	3.417	1237	750	1025	0.003	0.001	0.002
2	1	0.303	0.100	0.352	3.637	2.5	3.235	1091	750	971	0.002	0.001	0.002
3	1.25	0.268	0.100	0.328	3.210	2.5	3.016	963	750	905	0.002	0.001	0.002
4	1.5	0.236	0.100	0.301	2.833	2.5	2.767	850	750	830	0.002	0.001	0.002
5	1.75	0.208	0.100	0.272	2.500	2.5	2.500	750	750	750	0.002	0.001	0.002
6	2	0.184	0.100	0.242	2.206	2.5	2.224	662	750	667	0.001	0.001	0.001
7	2.25	0.162	0.100	0.212	1.947	2.5	1.947	584	750	584	0.001	0.001	0.001
8	2.5	0.143	0.100	0.183	1.718	2.5	1.678	515	750	504	0.001	0.001	0.001
9	2.75	0.126	0.100	0.155	1.516	2.5	1.424	455	750	427	0.001	0.001	0.001
10	3	0.112	0.100	0.130	1.338	2.5	1.190	401	750	357	0.001	0.001	0.001

by the use of prior distributions. Other parameter formulas are.

$$W_i = (\frac{PD_i}{PD_5}) \times 750 \tag{6.46}$$

$$Z_{2i} = (\frac{PD_i}{PD_5}) \times 2.5 \tag{6.47}$$

$$\lambda_i = (\frac{PD_i}{PD_5}) \times \lambda_5 \tag{6.48}$$

Input parameters values are listed in Table 3. Determination of the optimal model parameters is performed by minimizing loss cost under constraint through the R software package Optim. By using this package general-purpose optimization based on Nelder and Mead (1965), quasi-Newton and conjugate-gradient algorithms is done.

In Table 4,5 and 6, the comparison between optimal values and loss cost for economic statistical design by considering the non-uniform sampling and uniform sampling scheme for different values of the Weibull distribution shape parameter under three different loss functions (Linear, Quadratic, and Exponential) are given. As it is seen in Table 4,5 and 6 when we use economic statistical design, the loss cost becomes greater when uniform sampling scheme is used instead of non-uniform sampling scheme.

7. Conclusions

In this study, an approach was proposed for the design of a $\overline{X}$ control chart having economic and statistical properties for a process in which the failure mechanism obeys a Weibull shock model. This investigation mainly combined our cost model with Yang and Hancock's correlation model to develop an economic statistical design model of $\overline{X}$ charts for processes with correlated data. In addition, Three

Table 3: Optimal parameters obtained by considering Linear loss function.

		Economic Statistical design with non-uniform sampling scheme								Economic Statistical design with uniform sampling scheme							
k	PD	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$
	BR	6	1.46	2.99	0.013	0.85	1	0.47	157.7	5	1.39	2.94	0.014	0.82	0.99	0.49	163.31
1	NE	2	0.52	3	0.004	0.3	0.96	0.5	185	2	0.39	3	0.0025	0.3	0.99	0.49	29818.1
	Un	2	0.65	3	0.002	0.41	0.98	0.39	20140	2	0.5	3	0.004	0.41	1.01	0.83	112185
	HN	2	0.54	3	0.002	0.32	0.96	0.3	403	2	0.41	3	0.004	0.32	0.99	0.69	34746
	BR	3	3.03	2.11	0.049	0.74	0.99	0.43	151.76	20	1.98	3	0.081	0.99	1	0.46	313.77
1.5	NE	6	4.7	2.1	0.080	0.8	1	0.46	160	9	0.84	2.99	0.002	0.74	0.99	0.033	314.63
	Un	4	5.17	2.11	0.03	0.79	1	0.24	127.11	7	0.84	3	0.016	0.78	1	0.37	263.32
	HN	5	4.9	2	0.037	0.8	1	0.21	144.9	11	1.17	3	0.036	0.82	1	0.43	297.61
	BR	2	3.18	2.03	0.051	0.66	0.99	0.29	151.97	3	1.32	2.32	0.037	0.79	1	0.2	165.19
1.8	NE	4	5	2	0.08	0.73	0.99	0.23	141.09	12	1.12	3	0.001	0.81	1	0.013	308.55
	Un	3	5.97	2.04	0.037	0.76	1	0.13	115.86	5	0.9	2.43	0.036	0.78	1	0.44	216.03
	HN	4	5.22	2.06	0.034	0.74	0.99	0.1	130.5	6	0.77	2.75	0.024	0.72	0.99	0.28	259.9
	BR	2	3.27	2.05	0.049	0.65	0.98	0.2	149.16	4	1.37	2.36	0.036	0.81	1	0.14	166.69
2	NE	4	5.13	2.04	0.072	0.72	0.98	0.14	137.22	6	1.04	2.1	0.029	0.78	1	0.18	236.27
	Un	3	6.43	1.97	0.043	0.77	1	0.085	113.12	23	1.74	2.87	0.1	0.94	1	0.49	358.21
	HN	4	5.4	2.08	0.032	0.72	1	0.059	126.86	26	1.73	2.78	0.13	0.94	1	0.49	381.37
	BR	2	3.38	2.06	0.049	0.65	0.99	0.094	141.4	5	1.67	2.38	0.044	0.91	0.99	0.091	173.5
2.5	NE	2	5.85	1.2	0.25	0.78	1	0.092	128.06	6	1.06	2.12	0.027	0.78	1	0.1	229.43
	Un	3	6.43	1.92	0.05	0.77	1	0.027	114.09	25	1.85	2.38	0.2	0.97	1	0.5	361.63
	HN	4	5.8	1.86	0.05	0.77	1	0.02	123.62	27	1.72	2.94	0.12	0.94	1	0.27	370.41
	BR	2	3.32	2.02	0.053	0.67	0.99	0.049	138.61	24	2	3	0.098	0.99	1	0.11	296.23
3	NE	4	5.17	1.89	0.095	0.74	0.99	0.011	136.08	29	1.84	1.9	0.028	0.97	0.99	0.03	376.39
	Un	3	6.49	1.35	0.16	0.86	0.99	0.011	116.26	27	1.87	2.1	0.27	0.97	0.99	0.45	363.19
	HN	4	5.41	1.86	0.06	0.77	0.99	0.005	127.54	29	1.84	1.94	0.31	0.97	0.97	0.46	375.93
	BR	2	3	2	0.056	0.67	0.94	0.013	139.59	23	1.91	1.58	0.38	0.99	0.99	0.29	319.34
4	NE	7	4.94	0.47	0.7	0.97	0.98	0.004	150.8	29	1.87	1.67	0.048	0.97	0.99	0.041	352.67
	Un	4	5	2	0.039	0.81	0.94	0.007	135.4	28	1.89	1.75	0.36	0.98	0.98	0.37	354.51
	HN	6	5.03	0.54	0.47	0.97	0.99	0.003	141.21	4	1.18	1.83	0.11	0.81	0.99	0.19	202.55

Table 4: Optimal parameters obtained by considering Quadratic loss function.

		Economic Statistical design with non-uniform sampling scheme								Economic Statistical design with uniform sampling scheme							
k	PD	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$
	BR	6	1.46	3	0.014	0.85	1	0.47	148.28	6	1.46	3	0.013	0.84	1	0.46	148.16
1	NE	2	0.52	3	0.004	0.3	0.96	0.5	177.87	2	0.39	3	0.004	0.3	0.99	0.67	29904
	Un	2	0.65	3	0.004	0.41	0.98	0.64	20131	2	0.5	3	0.004	0.41	1	0.83	112176
	HN	2	0.54	3	0.004	0.32	0.96	0.51	395.39	2	0.41	3	0.004	0.32	0.99	0.68	34738
	BR	3	3.03	2.11	0.049	0.74	0.99	0.44	143.89	21	1.98	2.95	0.088	0.99	1	0.5	302.49
1.5	NE	6	4.77	2.14	0.084	0.8	1	0.46	153.37	9	0.84	3	0.024	0.74	1	0.4	307.46
	Un	4	5.17	2.11	0.062	0.79	1	0.5	119.01	7	0.84	3	0.01	0.77	1	0.37	255.27
	HN	5	4.9	2.01	0.088	0.8	1	0.49	137.52	11	1.17	3	0.03	0.82	1	0.44	290.11
	BR	2	3.18	2.03	0.052	0.66	0.99	0.3	144.85	20	1.98	3	0.079	0.99	1	0.27	279.6
1.8	NE	4	5	2	0.079	0.73	0.99	0.23	135.0	12	1.12	3	0.039	0.81	1	0.29	302.35
	Un	3	5.85	2.14	0.05	0.75	1	0.19	109.02	6	1.08	2.4	0.046	0.82	1	0.46	208.86
	HN	4	5.3	1.94	0.082	0.74	1	0.23	123.37	5	0.75	2.6	0.027	0.7	1	0.32	244.56
	BR	2	3.27	2.05	0.049	0.65	0.98	0.2	142.42	4	1.53	2.16	0.056	0.86	1	0.2	154.1
2	NE	3	5.12	2	0.072	0.72	0.98	0.14	131.99	27	1.68	2.82	0.13	0.93	1	0.5	392.36
	Un	3	5.91	2.24	0.08	0.73	0.1	0.095	109.08	23	1.74	2.87	0.1	0.94	1	0.5	351
	HN	3	5.67	1.88	0.09	0.75	0.99	0.14	119.44	26	1.73	2.78	0.13	0.94	1	0.49	375
	BR	2	3.38	3.06	0.049	0.65	0.99	0.093	135.3	5	1.58	2.39	0.04	0.88	1	0.089	157.66
2.5	NE	3	5.81	1.55	0.16	0.79	1	0.06	124.66	6	1.16	2.05	0.095	0.8	0.99	0.33	225.5
	Un	3	6.8	1.64	0.13	0.81	1	0.05	106.79	25	1.85	2.37	0.19	0.97	1	0.5	355.61
	HN	3	5.6	1.68	0.11	0.73	0.99	0.05	116.9	27	1.73	2.91	0.12	0.94	1	0.28	365.11
	BR	2	3.32	2.02	0.053	0.67	0.99	0.049	132.97	6	1.44	3	0.013	0.84	0.99	0.024	168.19
3	NE	2	6	0.47	0.64	0.91	0.99	0.02	120.19	29	1.84	1.92	0.32	0.97	0.99	0.46	372.05
	Un	2	6.49	1.07	0.3	0.86	0.99	0.02	108.31	27	1.88	2.1	0.26	0.98	0.99	0.45	357.6
	HN	3	5.9	1.21	0.27	0.87	0.99	0.013	118.31	29	1.84	1.94	0.31	0.97	0.97	0.46	371.29
	BR	2	3	2	0.056	0.67	0.94	0.013	134.52	3	1.4	2.19	0.049	0.83	1	0.5	160.72
4	NE	7	4.94	0.476	0.7	0.98	0.98	0.004	147.12	29	1.87	1.67	0.39	0.98	0.99	0.34	348.91
	Un	4	5	2	0.079	0.81	0.94	0.001	130.72	28	1.89	1.75	0.36	0.98	0.98	0.37	349.69
	HN	6	5.03	0.55	0.66	0.97	0.99	0.004	137.14	7	1.53	1.63	0.19	0.89	1	0.29	207.76

Table 5: Optimal parameters obtained by considering Exponential loss function.

		Economic Statistical design with non-uniform sampling scheme								Economic Statistical design with uniform sampling scheme							
k	PD	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$
	BR	6	1.46	3	0.013	0.84	1	0.46	165.95	6	1.46	3	0.013	0.85	1	0.47	165.94
1	NE	2	0.52	3	0.004	0.3	0.97	0.5	201	2	0.39	3	0.004	0.3	0.99	0.67	29844
	Un	2	0.65	3	0.004	0.41	0.98	0.64	20151	2	0.5	3	0.004	0.41	1	0.83	112195
	HN	2	0.54	3	0.004	0.32	0.96	0.51	417.99	2	0.41	3	0.004	0.32	1	0.68	34759
	BR	3	3.03	2.11	0.049	0.74	0.99	0.44	159.57	4	1.56	2.27	0.048	0.88	1	0.36	164.34
1.5	NE	6	4.77	2.14	0.084	0.8	1	0.47	170.34	9	0.84	3	0.024	0.74	1	0.4	324.66
	Un	4	5.17	2.11	0.062	0.79	1	0.49	136.06	7	0.84	2.99	0.016	0.77	1	0.38	272.65
	HN	5	4.9	2.01	0.088	0.79	1	0.49	154.54	11	1.17	3	0.036	0.82	1	0.43	307.03
	BR	2	3.17	2.04	0.052	0.66	0.99	0.29	159.8	20	1.99	3	0.078	0.99	1	0.27	294.39
1.8	NE	4	5	2	0.079	0.73	0.99	0.27	15.192	12	1.12	2.99	0.039	0.81	1	0.29	318.72
	Un	3	5.8	2.17	0.05	0.75	0.99	0.18	126.15	5	1.07	2.35	0.048	0.81	1	0.49	222.64
	HN	3	5.3	1.92	0.08	0.74	0.99	0.24	140.35	7	1.14	2.29	0.06	0.8	1	0.49	243.56
	BR	2	3.27	2.05	0.049	0.65	0.98	0.2	156.72	4	1.39	2.34	0.037	0.82	1	0.15	166.7
2	NE	4	5.1	2.03	0.073	0.72	0.97	0.14	149.21	29	1.68	2.82	0.13	0.92	1	0.5	407.2
	Un	3	6.09	2.04	0.06	0.74	1	0.13	123.5	23	1.74	2.87	0.1	0.95	1	0.49	365.98
	HN	4	5.42	2.07	0.06	0.72	0.99	0.11	137.79	4	0.83	2.1	0.06	0.71	1	0.49	233.64
	BR	2	3.39	2.06	0.049	0.65	0.99	0.09	149.02	6	1.73	2.44	0.045	0.93	0.99	0.087	176.42
2.5	NE	4	5.26	2	0.074	0.72	0.99	0.042	146.21	4	1.05	1.77	0.11	0.77	1	0.146	235.9
	Un	3	6.75	1.62	0.13	0.8	0.99	0.058	121.91	25	1.85	2.37	0.19	0.96	1	0.49	368.84
	HN	4	5.58	1.98	0.07	0.73	0.1	0.038	136.21	4	1.11	1.79	0.11	0.79	1	0.42	224.78
	BR	2	3.35	1.98	0.058	0.67	0.99	0.05	145.95	4	1.5	2.36	0.04	0.85	0.99	0.067	172.24
3	NE	4	5.17	1.89	0.095	0.75	0.99	0.01	148.09	29	1.83	1.92	0.032	0.97	0.98	0.46	386.35
	Un	2	6.7	0.64	0.53	0.92	0.99	0.02	121.09	27	1.87	2.1	0.26	0.97	0.98	0.45	370.45
	HN	4	5.54	1.71	0.1	0.79	0.99	0.01	137.2	29	1.85	1.94	0.31	0.97	0.98	0.46	383.75
	BR	2	3	2	0.056	0.67	0.94	0.01	146.93	4	1.45	2.47	0.03	0.84	1	0.036	175.12
4	NE	6	4.96	0.39	0.74	0.97	0.99	0.004	156.27	29	1.87	1.66	0.39	0.97	0.99	0.34	361.45
	Un	4	5	2	0.079	0.81	0.95	0.001	143.88	28	1.89	1.75	0.36	0.98	0.98	0.36	361.03
	HN	5	5.02	0.49	0.67	0.97	0.98	0.004	147.68	29	1.9	1.7	0.38	0.98	0.99	0.33	359.98

different loss functions (Linear, Quadratic, Exponential) were incorporated into our economic statistical design of $\overline{X}$ control charts by redefining the in-control and the out-of-control costs. The resulting model combines the advantages of the economic statistical design and loss function philosophy. In practice, multiple assignable causes are more realistic than the single ones. To provide more protection for both consumers and producers, an economic statistical model of $\overline{X}$ control chart integrated with three loss functions for correlated data under Weibull shock model with multiple assignable causes was proposed and $AATS$ and ANF have derived and calculated to determine the optimal design parameters. The economic statistical design with non-uniform and uniform sampling schemes are compared. Based on above comparison the cost model with non-uniform sampling cost has a lower cost than that with uniform sampling.

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