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the two formulas $\Box(\forall x)\phi_x$ and $(\forall x)\Box\phi_x$ therefore, he has accepted Barcan's formula and its converse. It is, we believe, safe to say that the Barcan formula must be called the «Sina - Barcan formula» (SBaF) and Buridan formula must be called the « Sina -Buridan formula » (SbuF) (Nabavi, 2000/ Movahed , 2002). As we see, in comparison with Aristotelian modal logic, the **ATM** theory arrived at a high degree of complexity. N. Rescher says:

> The Arabic logicians of Middle ages ... were in possession of a complex theory of temporal modal syllogisms ... when one considers that all reasoning was conducted purely, verbally, largely on the basis of somewhat vague examples, without any symbolic apparatus, and even without abbreviated devices. One cannot but admire the level of complexity and accuracy. (1974, p. 56)

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Budidan show the logical relations between modality-De re and De dicto as follows:

$(\forall x)\Box\ \phi_x \supset \Box\ (\forall x)\,\phi_x$	Barcan formula
$\Box\ (\forall x)\,\phi_x \supset (\forall x)\Box\,\phi_x$	converse of Barcan formula
$\Diamond\ (\forall x)\phi_x \supset (\forall x)\,\Diamond\,\phi_x$	Buridan formula
$(\forall x)\,\Diamond\ \phi_x \supset\ \Diamond\ (\forall x)\phi_x$	converse of Buridan formula

Through a detailed study, we see that Avicenna, a few centuries before « Jean Buridan»
(14th century) and «Marcus Ruth Barcan» (20th century) recognized this distinction. Avicenna says:

> Real modality either is located near the relation (copula) and in this case modality denotes and refers to quality of relation that assign predicate to object (subject) or is located near the universal or particular quantifier ... if we say «every human may (possibly) be a writer», the modality has natural situation and means all persons from human, one to one, may (possibly) be writer and if modality is located near the quantifier... it is modality of generalization and instantiation and we will arrive at different meaning like: all hummans (all together) may possibly be writer. The reason for this difference is that in the first case there is not any doubt every human (all one to one) may be a writer... but in the second case possibility is modality of universality and quantifier and it is a doubtful proposition, because it may be impossible that all humans (all together)are writers. (1952, p. 115)

In the above important paragraph, Avicenna obviously offers a counter example for the converse of Buridan's formula and therefore emphasizes the difference between the meaning of the two formulas $\Diamond(\forall x)\phi_xand(\forall x)\,\Diamond\,\phi_x$.

Avicenna also discusses the relation between necessity and universal quantifier. He says:

> But in necessity there is no difference between two modalities (quantifier modality and copula modality) ... such that every one is true, another is true too. (1960, p.170)

In the above paragraph, Avicenna emphasizes the equal meanings of

Table (10): Mixed valid moods of «AO-O»

major / Minor	□E	∀E	□C	∀C	□C + ~∀E	∀C + ~∀E
□C + ~∀E	∀E		∀C			
∀C + ~∀E	∀E		∀C			

Table (11): Mixed valid moods of «I E-O»

major / Minor	□E	∀E	□C	∀C	□C + ~∀E	∀C + ~∀E
□C + ~∀E	□E + ~∀E		∀C + ~∀E			
∀C + ~∀E	∀E + ~∀E		∀C + ~∀E			

In the fourth figure, out of a total of 7744 moods, 901 moods are valid and deduced and the remaining 6843 moods are invalid.

It must be noted that the above tables(No,3-11) were first shown by« Nasir al-Din al-Tusi» in his famous book « *Asas al-iqtibas fi'l-mantiq* » (pp. 221-248) and then by other logicians especially « Qutb al-Din al-Razi » in his two important books « *Sharh al-risalah al-shamsiyyah*» (pp.151-160) and « *Sharh matali al-anwar* » (pp.278-294).Without doubt,« Shirwani» in«*Sharh al-takmil fi'l-mantiq* » has taken these tables from« Qutb al-Din al-Razi» .

6. Avicenna and modality-De re and modality-De dicto

When we study the **ATM** theory, it is quite appropriate to discuss Avicenna's view of modality-De re and modality-De dicto. We know that one of the important problems in modern predicate modal logic is the relation between modality and quantifier. Today, based on the recent literature, if the scope of a modal operator (□, ◊) contains a formula with a free variable (open formula or propositional function), this modality is called «De re» and if instead there is no free variable (closed formula or proposition) in the scope of modality operator, this modality is called «Dedicto» (cf. Cresswell and Hughes, 1996, p. 250), therefore two formulas □ (∀x) Fx and ◊ (∃x) Fx have De dicto modality and (∀x)□ Fx and (∀x)◊ Fx have De re modality. Also we know that the famous formulas i.e., Barcan, converse of Barcan, Buridan and converse of

Table (8): Mixed valid moods of «Camenes» (AE-E)

<table>
<tr><th>major / minor</th><th>□E</th><th>∀E</th><th>□C</th><th>∀C</th><th>□C + ~∀E</th><th>∀C + ~∀E</th><th>remaining actual</th></tr>
<tr><td>□E</td><td colspan="7" rowspan="2">∀E</td></tr>
<tr><td>∀E</td></tr>
<tr><td>□C</td><td colspan="6" rowspan="2">∀C</td><td rowspan="4">(invalid)</td></tr>
<tr><td>∀C</td></tr>
<tr><td>□C + ~∀E</td><td colspan="2" rowspan="2">∀E</td><td colspan="4" rowspan="2">(∀C) + (~∀E)</td></tr>
<tr><td>∀C + ~∀E</td></tr>
</table>

In Table (9), (∀C) + (~∀E) refers to «non-perpetual - about some conventional» (*orphiyyah la Daemata fi'l - Baaz*) (Rescher, 1974, p. 31).

In the fourth figure, when we discuss Avicennan temporal modalities, we must know the three other moods i.e., «OA-O» ، «AO-O» and «IE-O» also are valid. The reason for this matter, briefly, is that the particular negative (O) of two specials i.e., special conditional (□ C + ~∀E) and special conventional (∀C + ~∀E) is convertible (simple conversion), and consequently the number of the valid moods in the fourth figure is increased to eight. (Razi, Qutb al-Din, 1984, pp.157-159/ Rescher, 1974, PP. 43-48). The following tables show valid moods of these three additional moods.

Table (9): Mixed valid moods of «OA-O»

<table>
<tr><th>major / Minor</th><th>□C + ~∀E</th><th>∀C + ~∀E</th></tr>
<tr><td>□E</td><td colspan="2" rowspan="8">∃C + ~∀E</td></tr>
<tr><td>∀E</td></tr>
<tr><td>□C</td></tr>
<tr><td>□C + ~∀E</td></tr>
<tr><td>∀C</td></tr>
<tr><td>∀C + ~∀E</td></tr>
<tr><td>∃C</td></tr>
<tr><td>∃C + ~∀E</td></tr>
<tr><td>□T</td><td colspan="2">∃E</td></tr>
<tr><td>remaining actual</td><td colspan="2">∃E + ~∀E</td></tr>
</table>

therefore, we must consider first valid moods regarding quantity and quality (i.e., Bramantip, Camenes, Fesapo, Fresison and Dimaris) and then introduce deduction conditions corresponding to one to one or two to two of these moods. The following tables show only the mixed valid moods based on the above five syllogistic forms.

Table (6): Mixed valid moods of «Bramantip» (AA-A) and «Dimaris» (IA-I)

<table>
<tr><th>Major / Minor</th><th>□ E</th><th>∀E</th><th>□ C</th><th>∀C</th><th>□ C + ~∀E</th><th>∀C + ~∀E</th><th>remaining actual</th></tr>
<tr><td>□ E</td><td colspan="6" rowspan="4">∃C</td><td rowspan="2">∃C</td></tr>
<tr><td>∀E</td></tr>
<tr><td>□ C</td><td rowspan="5">∃E</td></tr>
<tr><td>∀C</td></tr>
<tr><td>□ C + ~∀E</td><td colspan="6" rowspan="2">∃C + ~∀E</td></tr>
<tr><td>∀C + ~∀E</td></tr>
<tr><td>remaining actuals</td><td colspan="6">∃E</td></tr>
</table>

Table (7): Mixed valid moods of «Fesapo» (EA-O) and «Fresison» (EI-O)

<table>
<tr><th>major / Minor</th><th>□ E</th><th>∀E</th><th>□ C</th><th>∀C</th><th>□ C + ~∀E</th><th>∀C + ~∀E</th></tr>
<tr><td>□ E</td><td colspan="2" rowspan="9">∀E</td><td colspan="4" rowspan="6">∃C</td></tr>
<tr><td>∀E</td></tr>
<tr><td>□ C</td></tr>
<tr><td>□ C + ~∀E</td></tr>
<tr><td>∀C</td></tr>
<tr><td>∀C + ~∀E</td></tr>
<tr><td>∃C + ~∀E</td><td colspan="2"></td><td colspan="2" rowspan="2">∀C</td></tr>
<tr><td>∃C</td><td colspan="2"></td></tr>
<tr><td>remaining actual</td><td colspan="4">∃E</td></tr>
</table>

quality are affirmation of the minor and universality of at least one of the premises and regarding modality it is actuality of the minor (like the first figure). Table (5) shows all the valid and invalid moods in the third figure.

Table (5): Mixed moods in the third figure

<table>
<tr><th>major / minor</th><th>□ C</th><th>∀C</th><th>□ C + ~∀E</th><th>∀C+ ~∀E</th><th>18 rested propositions in the minor column</th></tr>
<tr><td>□ E</td><td colspan="2" rowspan="8">∃C</td><td colspan="2" rowspan="8">∃C + ~ ∀E</td><td rowspan="17">(Valid)
same as entry at the major</td></tr>
<tr><td>∀E</td></tr>
<tr><td>□ C</td></tr>
<tr><td>□ C + ~ ∀E</td></tr>
<tr><td>∀C</td></tr>
<tr><td>∀C + ~ ∀E</td></tr>
<tr><td>∃C</td></tr>
<tr><td>∃C + ~ ∀E</td></tr>
<tr><td>□ T</td><td colspan="2" rowspan="9">∃E</td><td colspan="2" rowspan="9">∃E + ~ ∀E</td></tr>
<tr><td>□ T + ~ ∀E</td></tr>
<tr><td>□ S</td></tr>
<tr><td>□ S + ~ ∀E</td></tr>
<tr><td>∃E</td></tr>
<tr><td>∃E + ~ ∀E</td></tr>
<tr><td>∃E + ~ □ E</td></tr>
<tr><td>∃T</td></tr>
<tr><td>∃S</td></tr>
<tr><td>◊ E</td><td colspan="5" rowspan="5">Invalid</td></tr>
<tr><td>◊ E + ~ □ E</td></tr>
<tr><td>◊ C</td></tr>
<tr><td>◊ T</td></tr>
<tr><td>◊ C</td></tr>
</table>

In the third figure, out of a total of 7744 moods, 2244 moods are valid and deduced and the remaining 5500 moods are invalid.

4. In the fourth figure, there is not a general deduction condition

E or both ∀E). Table (4) shows all the valid and invalid moods in the second figure.

Table (4): Mixed moods in the second figure

<table>
<tr><th>major / minor</th><th>□C</th><th>□C + ~ ∀E</th><th>∀C</th><th>∀C + ~∀E</th><th>□ E</th><th>∀E</th><th>another propositions</th><th>Possible propositions</th></tr>
<tr><td>□ E</td><td colspan="8">∀E</td></tr>
<tr><td>∀E</td><td colspan="7"></td><td rowspan="21">invalid</td></tr>
<tr><td>□ C</td><td colspan="4" rowspan="4">∀C</td><td colspan="2" rowspan="15">∀E</td><td rowspan="20"></td></tr>
<tr><td>□ C + ~ ∀E</td></tr>
<tr><td>∀C</td></tr>
<tr><td>∀C + ~ ∀E</td></tr>
<tr><td>□ T</td><td colspan="4" rowspan="3">∃T</td></tr>
<tr><td>∃T</td></tr>
<tr><td>□ T + ~ ∀E</td></tr>
<tr><td>□ S</td><td colspan="4" rowspan="3">∃S</td></tr>
<tr><td>∃S</td></tr>
<tr><td>□ S + ~ ∀E</td></tr>
<tr><td>∃E</td><td colspan="4" rowspan="3">∃E</td></tr>
<tr><td>∃E + ~ ∀E</td></tr>
<tr><td>∃E + ~ ∀E</td></tr>
<tr><td>∃C</td><td colspan="4" rowspan="2">∃C</td></tr>
<tr><td>∃C + ~ ∀E</td></tr>
<tr><td>◊ E</td><td colspan="3" rowspan="2">◊ E</td><td rowspan="5">Invalid</td><td rowspan="5">∀E</td><td rowspan="5"></td></tr>
<tr><td>◊ E+ ~ □ E</td></tr>
<tr><td>◊ C</td><td colspan="3">◊ C</td></tr>
<tr><td>◊ T</td><td colspan="3">◊ T</td></tr>
<tr><td>◊ S</td><td colspan="3">◊ S</td></tr>
</table>

From a total of 7744 moods in the second figure, 576 moods are valid and deduced and 7168 moods are invalid.

3. In the third figure, deduction conditions regarding quantity and

∃E + ~ ∀E			
∃E + ~ □ E			
∃C	∃C	∃C + ~ ∀E	
∃C + ~ ∀E			
∃T	∃T	∃T + ~ ∀E	
∃S	∃S	∃S + ~ ∀E	
◊ E			
◊ E + ~ □ E			
◊ C		Invalid	
◊ T			
◊ S			

From the whole 7744 moods in the first figure (regarding quantity, quality and modality), 1469 moods are valid and deduced and 6263 are invalid.

2. In the second figure, deducing conditions regarding quantity and quality are the difference of premises in negation and affirmation and universality of the major and regarding modality they are the two following states :

A) Perpetuity of the minor (provided that if the major is possible, the minor must be absolute necessary i.e., □ E)

B) Being « negative convertible» of the major (provided that if the minor is possible, the major must be one of « three necessaries») (pp.151-152).

About these conditions it must be noted, firstly, that both □ E and ∀E are perpetual; secondly, that « negative convertible» propositions are six temporal modalities (□ E , □ C , ∀E , ∀C ، □ C + ~ ∀E and ∀C + ~ ∀E) i.e., the six propositions whose universal negative are convertible, including two absolutes (absolute necessary and absolute perpetual), two generals (general conditional and general conventional) and two specials (special conditional and special conventional); thirdly, that« three necessaries» are referred to three propositions i.e., □E , □C and □C + ∀E ; and fourthly, that the above two states of deduction conditions are inclusive and not exclusive because these two conditions are compatible with each other (for example when the minor and major can be both □

invalid) are calculated through the following formula:

moods = (4 quarter quantified ×22 temporal modalities)$^{2\ \text{premises}}$ × 4 figures

$$\text{moods} = (4 \times 22)^2 \times 4 = 30{,}976$$

From 30,976 moods (7744 moods in every figure) in **ATM** theory, 5112 moods are valid or deduced in all four figures and the rest (25,864 moods) are invalid or no deduced.

1. In the first figure, deduction (or validity) conditions regarding logical quantity and quality are affirmation of the minor and universality of the major and regarding modality it is actuality of the minor (Razi, Qutb al-Din , 1984, pp.149-150).

Actual propositions are modal propositions with □, ∀ and ∃ operators except possible propositions (i.e., ◊ E , ◊ C , ◊ T , ◊ S and ◊ E + ~ □ E). Therefore the number of actual propositions in **ATM** theory are seventeen. Table (3) introduces all the valid and invalid moods and their conclusion in **ATM** theory.

Table (3): Mixed moods in the first figure

<table>
<tr><th>major / minor</th><th>□ C</th><th>∀C</th><th>□ C + ~ ∀E</th><th>∀C + ~ ∀E</th><th>18 rested propositions in the minor column</th></tr>
<tr><td>□ E</td><td>□ E</td><td rowspan="2">∀E</td><td>□ E + ~ ∀E</td><td rowspan="2">∀E + ~ ∀E</td><td rowspan="12">Same as entry at the major</td></tr>
<tr><td>∀E</td><td>∀E</td><td>∀E + ~ ∀E</td></tr>
<tr><td>□ C</td><td rowspan="2">□ C</td><td rowspan="2">∀C</td><td rowspan="2">□ C + ~ ∀E</td><td rowspan="2">∀C + ~ ∀E</td></tr>
<tr><td>□ C + ~ ∀E</td></tr>
<tr><td>∀C</td><td colspan="2" rowspan="2">∀C</td><td colspan="2" rowspan="2">∀C + ~ ∀E</td></tr>
<tr><td>∀C + ~ ∀E</td></tr>
<tr><td>□ T</td><td rowspan="2">□ T</td><td rowspan="2">∃T</td><td rowspan="2">□ T + ~ ∀E</td><td rowspan="2">∃T + ~ ∀E</td></tr>
<tr><td>□ T + ~ ∀E</td></tr>
<tr><td>□ S</td><td rowspan="2">□ S</td><td rowspan="2">∃S</td><td rowspan="2">□ S + ~ ∀E</td><td rowspan="2">∃S + ~ ∀E</td></tr>
<tr><td>□ S + ~ ∀E</td></tr>
<tr><td>∃E</td><td colspan="2">∃E</td><td colspan="2">∃E + ~ ∀E</td></tr>
</table>

Aristotelian categorical logic we have sixty four syllogistic moods in total, out of which nineteen moods are valid and are the following:

Figure (I): Barbara (AA-A) - Celarent (EA-E) - Darii (AI-I)- Ferio (EI-O)

Figure (II): Cesare (EA-E) - Camestres (AE-E) - Festino (EI-O)- Baroco (AO-O)

Figure (III): Darapti (AA-I) - Datisi (AI-I) - Disamis (IA-I) - Felapton (EA-O) - Ferison (EI-O) - Bocardo (OA-O)

Figure (IV): Bramantip (AA-I) - camenes (AE-E) - Fesapo (EA-O) - Fresison (EI-O) –

Dimaris (IA-I) (Lukasiewicz, 1972, pp. 92-93).

It must be noted that Al-Farabi and then Avicenna, in contrast with Aristotle and the later Greek-Latin tradition, believed that in the structure of a syllogism we must first mention the minor premise and then the major one. They believed in this way, although it is different from Aristotelian syntax and formalization, it preserves the logical semantic and meaning of Aristotelian logic precisely. We know in Greek the predicate is mentioned naturally before the subject (B is predicated of all A = το B κατα παντος A). Therefore, Aristotle has mentioned the major premise before the minor in order to preserve the linguistic connection and consequently self-evidence of the first figure as follows:

« **τὸ Γ κατὰ παντὸς του Β καὶ τὸ Β κατὰ παντὸς του Α, , ανάγκη τὸ Γ κατὰ παντὸς του Α**» (Aristotle, *Analytica priora,* 1949, 4 (25b, 39-41).

But in Latin, Arabic and Persian, however, the subject must naturally be mentioned before the predicate, in which case obedience to the Aristotelian formal tradition obscures the logical connection and therefore the self evidence of the first figure.Al-Farabi and Avicenna intelligently changed this tradition and with the commutation of the minor and major premises preserved the Aristotelian logical semantic and meaning as is shown below:

« **All A is B and all B is C, therefore All A is C**»

(Farabi, Abu Nasr , 1988**,** vol.1, p.129).

Within the scope of the **ATM** theory, we concentrate on all the possible moods in the categorical syllogism. Based on Shirwani's version of 22 temporal modalities، the number of all syllogistic moods (valid or

The famous thirteen modal propositions that are mentioned by «Qutb al-Din al-Razi» in «*Sharh al-shamsiyyah*» are:

1- two absolutes (*motlaghatan*)	absolute necessary ($\Box E$)
	absolute perpetual ($\forall E$)
2- two generals (*aamatan*)	general conditional ($\Box C$)
	general conventional($\forall C$)
3- two possibles (*momkenatan*)	general possible ($\Diamond E$)
	special possible ($\Diamond E + \sim \Box E$)
4- two specials (*khassatan*)	special conditional ($\Box C + \sim \forall E$)
	special conventional ($\forall C + \sim \forall E$)
5- two existentials (*wojowdiyatan*)	non-perpetual existential ($\exists E + \sim \forall E$)
	non-necessary existential ($\exists E + \sim \Box E$)
6- two temporals (*waghtiyatan*)	temporal ($\Box T + \sim \forall E$)
	spread ($\Box S + \sim \forall E$)
7- and general absolute (*motlageh aameh*) ($\exists E$)	

Through careful examination of Table (2), we find the most important compound proposition is «special possible» ($\Diamond E + \sim \Box E$) that Aristotle had called «Contingent» (ενδεχομενον). This proposition has a fundamental role in the generation of **ATM** theory and its development. If we also consider the similar names like «special possible» (*momkeneh khasseh*), « special conditional » (*mashroteh khasseh*) and «special conventional» (*orphieh khasseh*) in the table above, this role is clearly obvious.

5. Deductive apparatus of ATM theory

Avicenna and his followers, based on formal language of **ATM** theory (Tables 1 and 2), have devised a complex and sophisticated deductive apparatus especially in conversion and syllogism. In this section, we briefly introduce the **ATM** syllogistic system and show the efficiency of Rescher's formulation for logical calculations in **ATM** theory. In

				essential absolute is not B
4	(□S + ~ ∀E)	spread	non-perpetual spread necessary	All A, of spread necessity is B and of essential absolute is not B
5	(∀E + ~ ∀E)	non-perpetual perpetual	non-perpetual essential perpetual	All A, of essential perpetuity is B and of essential absolute is not B
6	(∀C + ~ ∀E)	special conventional	non-perpetual conditional perpetual	All A, of conditional perpetuity is B and of essential absolute is not B
7	(∃E + ~ ∀E)	non-perpetual existential	non-perpetual essential absolute	All A, of essential absolute is B and of essential absolute is not B
8	(∃E + ~ □E)	non-necessary existential	non-necessary essential absolute	All A, of essential absolute is B and of essential possibility is not B
9	(∃C + ~ ∀E)	non-perpetual absolute continuing	non-perpetual conditional absolute	All A, of conditional absolute is B and of essential absolute is not B
10	(∃T + ~ ∀E)	non-perpetual temporal absolute	non-perpetual temporal absolute	All A, of temporal absolute is B and of essential absolute is not B
11	(∃S + ~ ∀E)	non-perpetual spread absolute	non-perpetual spread absolute	All A, of spread absolute is B and of essential absolute is not B
12	(◊ E + ~ □E)	special possible	non-necessary essential possible	All A, of essential possibility is B and of essential possibility is not B

developed by Avicenna and his followers, only some of these propositions have been treated in detail. For example, in the most complete version and development of **ATM** theory i.e., Shirwani's version in his book « *sharh al-takmil fi'l-mantiq*» (Rescher, 1974), we see that only twenty two modal propositions (14 simple & 8 compound) are used in logical calculations i.e., in conversion, opposition and especially modal syllogisms. It also must be noted that some compound forms are identical, for example ($\Diamond E + \sim \forall E$) ≡ ($\exists E + \sim \Box E$).

Some of the later Muslim logicians like «Muhammad Forsat Shirazi» (19th century) have proposed a more complicated theory by introducing other restrictions like « non-conditional perpetuity» ($\sim \forall C$) and «non-conditional necessity» ($\sim \Box\ C$) for the formation of compound propositions (Shirazi, Forsat, 1993, pp.72-73).

In most logical texts using **ATM** theory (except Shirwani's book), only thirteen modal propositions (six simple & seven compound) provide the bases of logical calculations. «Qutb al-Din al-Razi» one of the famous commentators of **ATM** theory says:

> The number of simple and compound propositions is not limited to a certain number but what is customary and usual is that only thirteen propositions are used in contradiction, conversion and syllogism. (Razi, Qutb al-Din, 1984, p.103)

Table (2) introduces twelve compound propositions as «Shirwani» has discussed.

Table (2) : Compound modal propositions in ATM theory

	Code	ordinary name	Structural name	logical from (A)
1	($\Box E + \sim \forall E$)	non-perpetual necessary	non-perpetual essential necessary	All A, of essential necessity is B and of essential absolute is not B
2	($\Box C + \sim \forall E$)	special conditional	non-perpetual conditional necessary	All A, of conditional necessity is B and of essential absolute is not B
3	($\Box T + \sim \forall E$)	temporal	non-perpetual temporal necessary	All A , of temporal necessity is B and of

12	◊C	possible continuing	conditional possible	All A of conditional possibility is B	All writers move with a possibility while they are writing.
13	◊T	temporal possible	temporal possible	All A of temporal possibility is B	The moon is eclipsed with a possibility at the time when the earth is between it and the sun.
14	◊S	perpetual possible	spread possible	All A of spread possibility is B	All men breathe with a possibility at some times.

It must be noted that the above ordinary textual names , used in all post-Avicennan logical texts, were first offered by« Fakr al-Din al-Razi» (Razi, Fakr al-Din, 2002, p.169/ Tusi, Nasir al-Din,, 1960,p.165). In Table(1), the translation of «ordinary names» and «well-known paradigm examples» from the Arabic language to English (with a few correctness) are all from Nicholas Rescher (1974, pp.22-23).

If the predicate or predicate part of a categorical proposition contains two temporal modalities (twofold predicative), one affirmative and another negative, it is named a «compound» modal proposition. In natural language, compound propositions are formed by adding two restrictions at the end of a proposition. These restrictions can only take one of the following two forms:

1. non-necessity (essential non-necessity), for example : All men write with possibility, but not necessity.
2. non-perpetuity (essential non-perpetuity), for example: All writers move of necessity, but not perpetuity

In this article, we designate 'non-necessity' with the symbolic code «$\sim\Box E$» and 'non-perpetuity' with «$\sim\forall E$ ». In **ATM** theory we also have the two following equations:

$$(\sim\Box\, E)\, P = (\Diamond\, E) \sim P$$
$$(\sim \forall E\,)\, P = (\,\exists E\,) \sim P$$

By combining fourteen simple propositions and the above two restrictions ($\sim \Box E$, $\sim\forall E$), we can make twenty eight compound propositions, which when added to the fourteen simple ones, will give us forty two (42) modal propositions, but in the **ATM** theory, as it is

Table(1): simple modal propositions in ATM theory

	code	ordinary name	structural name	logical form (A)	textual example
1	□E	absolute necessary	essential necessary	All A of essential necessity is B	All men are rational of necessity as long as they exist.
2	□C	general conditional	conditional necessary	All A of conditional necessity is B	All writers move their fingers of necessity as long as they write.
3	□T	absolute temporal	temporal necessary	All A of temporal necessity is B	The moon is eclipsed of necessity at the time when the earth is between it and the sun.
4	□S	absolute spread	spread necessary	All A of spread necessity is B	All men breathe of necessity at some times
5	∀E	absolute perpetual	essential perpetual	All A of essential perpetuity is B	All men are rational perpetually as long as they exist.
6	∀C	general convention al	conditional perpetual	All A of conditional perpetuity is B	All writers move as long as they write.
7	∃E	general absolute	essential absolute	All A of essential absolute is B	All men breathe at some times as long as they exist.
8	∃C	absolute continuing	conditional absolute	All A of conditional absolute is B	All writers move while they are writing.
9	∃T	temporal absolute	temporal absolute	All A of temporal absolute is B	All writers move at the time they are writing.
10	∃S	spread absolute	spread absolute	All A of spread absolute is B	All men breathe at some times
11	◊E	general possible	essential possible	All A of essential possibility is B	All writers move with a possibility as long as they exist.

2. *ma dam al-wasf* (with code C) : during times of existence of the subject with the condition of subject property

3. *fi waghten moayan* (with code T) : during a certain specified and determinate period of the existence of the subject

4. *fi waghten ghair moayan* (with code S) : during some unspecified and indeterminate period of the existence of the subject

The above symbolic codes for temporal operators i.e., ∀ and ∃ and temporal restrictions i.e., E ، C، T and S are all from Rescher (1974, pp.21-22). The propositions that contain these four restrictions (E، C، T ، S)are named as «essential» (or existential), « conditional », «temporal» and «spread» respectively (Tusi, 1984, pp.65-67). By the combination of four modal and temporal operators (□, ∀ , ∃, ◊) , on the one hand, and four temporal restrictions (E, C, T, S), on the other, we can obviously make sixteen «temporal modalities » (Avicenna, 1960, p.145) as follows:

1- □E	5- ∀E	9- ∃E	13- ◊E
2- □C	6- ∀C	10- ∃C	14- ◊C
3- □T	7- ∀T	11- ∃T	15- ◊T
4- □S	8- ∀S	12- ∃S	16- ◊S

From the above mentioned sixteen temporal modalities, two forms i.e., « ∀T » and « ∀S » are meaningless and therefore not used in **ATM** theory (Tusi, 1982, pp.131-133).

Simple and Compound Modalities

Avicenna and his followers have divided modal propositions into two general groups: simple and compound propositions. A simple modal proposition is one whose predicate or predicative part contains only one temporal modality.

Based on the above definition and keeping in mind that there are fourteen temporal modalities, we will make fourteen simple propositions. In Table (1), we introduce the code, ordinary textual name, structural name, logical form and a well-known textual example(paradigm example) for each simple modal proposition.

structure of a categorical proposition.

quality (*poion*) {
1. permanent property (*poiotes*)
2. enduring state (*schesis*)
3. transient characteristic (*hexis*)

The exact distinction among the above three types of quality related to the temporal interpretation of a proposition that contains these qualities is as explained below (pp.5051).

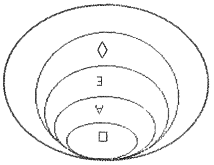
1. human is animal all the time (permanent property)
2. a prudent human acts wisely most of the time (enduring state)
3. a healthy human walks some time (transient characteristic)

Without any doubt , as Rescher emphasizes, the above temporal predicates in Stoic texts are the roots of the following Avicennan concepts in **ATM** theory.

1.as long as the essence exists (*ma dam al-zat*)
2. as long as the property exists (*ma dam al-wasf*)
3. as long as the time (certain or uncertain) (*ma dam al-waght*)

4. Formal language of ATM theory

Avicenna combines two modal concepts , necessity (□) and possibility (◊) with two temporal notions perpetuity (with code ∀) and actuality (with code ∃). The meaning of actuality (same «absolute» in the *organon*) is to occur at one time and perpetuity (opposite of actuality) is to occur at all times. Necessity has the narrowest scope, possibility has the broadest and perpetuity and actuality are between these two concepts as shown below:



In **ATM** theory, the propositions that contain these four operators (□ ,∀ , ∃, ◊) are named 'necessary' (*Zarorieh*), 'perpetual' (*Daemeh*), 'actual' (*Motlageh*) and 'possible' (*Momkeneh*) respectively (Avicenna, 1960, pp.143-144). Avicenna, also defines four temporal restrictions (similar to Stoic temporal predicates) as follows:

1. *ma dam al-zat* (with code E) : during times of existence of the essence of the subject

writes:

> The founder of logic (Aristotle) in *Analytica priora* said: the propositions are of three types: necessary, possible and absolute , and his commentators have different opinions in the interpretation of "absolute". The view of « Themistius » and « Theophrastus» is that the absolute proposition is general absolute and others such as « Aphrodisias » later on have said essential necessary is not absolute and another actual propositions are absolute. (1982, pp.140-141)

Based on Aristotle's commentator's views, the concept of «huparchen » is interpreted obviously as having two meanings. One is in contrast with modality (a proposition without modality and free of it) and another as a type of modality similar to necessity and possibility. Avicenna uses both of the above meanings of absolute, but he emphasizes the second meaning (absolute as a modality) and has developed it in **ATM** theory. Based on the second meaning, a proposition is divided as follows:

- proposition
  - modal
    - actual
      - necessary
      - absolute (huparchen)
    - non-actual
      - possible
      - contingent
  - non-modal

3.4. Stoic-megarian temporal predicates: In addition to Aristotelian modal concepts, Stoic-Megarian logical views especially « temporal predicates» have had a fundamental role in the generation of **ATM** theory. About this matter, « Nicholas Rescher» writes:

> The notions of temporalized modality that are at work here are mainly those relating to the « master argument » of Diodorus Cronus ... In all of this there is no sign of the ramified machinery of temporalized modalities which we find in Arabic texts-and which are unquestionably of Greek provenience. For the roots of this theory we must undoubtedly look to the stoic doctrine of predication. (1974, PP. 49-50)

The stoic recognized the following three types of quality in the particular categorical propositions are as follows:

A: $(\forall x)\ (Ax \supset \Box Bx)$

A: $(\forall x)(Ax \supset \Diamond Bx)$

I: $(\exists x)\ (Ax \wedge \Box Bx)$

I: $(\exists x)\ (Ax \wedge \Diamond Bx)$

It is obvious that the above formulation is based on a De re-interpretation of modal propositions (De re- predicate).

Avicenna and his followers used these two Aristotelian modalities ($\Box$, $\Diamond$) clearly and completely in the construction of **ATM** theory.

3.2. Aristotelian Contingency: The concept of «contingency» (ενδεχεται) in Aristotelian modal logic (*Ibid*, 13 (32a, 18-21)) has had a fundamental role in the generation of **ATM** theory. In new notations, Aristotle defines it as follows:

$\nabla\phi =df\ \Diamond \phi \wedge \Diamond \sim \phi$ (bilateral possibility)

$\nabla\phi =df \sim \Box \sim \phi \wedge \sim \Box \phi$

In a categorical proposition, the contingency operator (∇) based on the De re- interpretation can be shown as follows:

$(\forall x)\ (Ax \supset \nabla Bx)$

$(\forall x)\ [Ax \supset (\Diamond Bx \wedge \Diamond \sim Bx)]$

3.3. *Aristotelian* absolute and actuality: The logical concept of «huparchen» (υπαρχεν) in Aristotle's *organon* is another important concept that has had a direct influence on the generation of **ATM** theory. Aristotle maintains:

> every premise states that something either is (applies, belongs) or must be (necessarily applies, must belong) or maybe (possibly applies, may belong) the attribute of something else. (Aristotle, *Analytica priora,* 1949, 2(25a, 1))

The earlier Syriac-Arabian translators of Aristotle's books like « Theodore » (Tadhari) (750-850) have translated the word « huparchen » as «absolute » (*Motlageh*) (Badawi, 1980, p.143).

The above concept and its logical interpretation has not been discussed in Latin texts, but Muslim logicians based on different views of Greek's commentators like «Theophrastus» and «Themistius» on the one hand and « Aphrodisias » on the other have treated this Aristotelian concept in detail.

«Nasir al-Din al-Tusi » one of the famous commentators of Avicenna

Avicenna's view and

post-Avicennan theories in this tradition.

(a): ***Kitab al-shifa*** (al-mantiq), Avicenna

(b): ***Kitab al-isharat wa'l-tanbihat (al-mantiq)***, Avicenna

(c): ***Al-mantiq al mulakhhas***, Fakr al Din al-Razi

(d): ***Sharh al-mantiq al-isharat***, Nasir al-Din al-Tusi

(e): ***Asas al-iqtibas fi'l-mantiq,*** Nasir al-Din al-Tusi

(f): ***Kitab al-tajrid fi'l-mantiq***, Nasir al-Din al-Tusi

(g): ***Al-risalah al-shamsiyyah fi'l-qawaid al-mantiqiyyah,*** Najm al-Din al-Katibi al-Qazwini

(h): ***Matali al-anwar fi'l-mantiq,*** Siraj al-Din al-Urmawi

(i): ***Sharh al-risalah al-shamsiyyah,*** Qutb al-Din al-Razi

(j): ***Sharh matali al-anwar,*** Qutb al-Din al-Razi

(k): ***Sharh al-takmil fi'l-mantiq,*** Muhammad al-Shirwani

3. Background

Through a meticulous study, we find the conceptual grounds and original ideas of **ATM** theory have existed in Aristotelian, Stoic and Megarian logical texts. We believe in a general perspective, the four following concepts have been of influence in the generation of **ATM** theory.

3.1. Aristotelian necessity and possibility*:* The logical concepts of « necessity» (αναγκης), which is by definition « negation of possibility of negation » and « possibility» (*δυνατο*),which is defined as «negation of necessity of negation » are two important concepts in Aristotelian modal logic (Aristotle, *De-Interpretation*, 1949, 12 (21 a, 35-36). In modern notation we have:

$$\Box \phi = df \sim \Diamond \sim \phi$$
$$\Diamond \phi = df \sim \Box \sim \phi$$

Also, we know Aristotle believed that these two modal operators (□ , ◊) are the property of the relation between subject and predicate or copula.

If we accept A. Becker and N. Rescher's investigations about description and interpretation of Aristotelian modal operators in a syllogism (McCall ,1963, p. 21/ Van Rijen, 1989, pp. 187-188), especially when the minor premise is without modality and major premise is necessary and its conclusion is necessary too (□×- □) (Aristotle, *Analytica priora*, 1949, 9 (30a, 15-19)), then the logical structure of universal and

-Fakr al-Din al-Razi(1149-1209)

- Nasir al-Din al-Tusi (1201-1274)
- Najm al-Din al-Katibi al-Qazwini (1220-1292)
- Siraj al-Din al-Urmawi (1198-1283)
- Qutb al-Din al-Razi (1290-1365)
- Shihab al-Din al-Suhrawardi (1155-1191)
- Muhamad al-Shirwani (early 15th century) (Rescher, 1964 and 1974)

Among these innovations, we must refer in particular to the following two important theories:

- Theory of conditional attributive (*Eghtrani*) syllogism
- Theory of temporal modalities (**ATM**)

Although, the basic concepts and elements of the above theories are founded in Aristotelian and Stoic-Megarian topics, these theories as independent logical systems are devised only by Avicenna just as he himself emphasized that he pioneered this subject (Avicenna, 1960, p.235). These two theories and their expansion and development especially after Avicenna are such that we can refer to this historical period as «Avicennan logic» and regard its historical importance comparable to other logical schools like Aristotelian and Stoic-Megarian logic.

"Nicholas Rescher" writes especially about the Avicennan theory of temporal modalities:

> Clearly the Arabic logicians of the middle ages - basing their work upon Greek antecedents - were in possession of complex theory of temporal modal syllogisms, which they elaborated in great and sophisticated detail ... . It is, I believe, safe to say that ... the logical theory of temporal concept was carried to a higher point in Arabic logic than at any subsequent juncture prior to our own times . (1974, p. 56).

In this article, with a historical and comparative approach, we study the theory of temporal modalities(**ATM**) based on the logical heritage of Avicenna and later logicians in such a way that we can show their relations with modern logical concepts.

2. Sources

The following sources are the most important logical texts that contain

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Time and Modality in Avicennan Logic

Lotfollah Nabavi[*]

Abstract
*One of the most important innovations in the history of traditional logic is the Avicena's Theory of Temporal Modalities (**ATM**). Although, the basic concepts and elements of this theory are founded in Aristotelian and Stoic-Megarian logic, but as a independent logical system, are devised only by Avicenna and were later developed and completed by his followers. The **ATM** theory contains the highest degree of logical complexity in the all periods of the traditional logic . In this article, with a historical and comparative approach, we study the theory of temporal modalities(**ATM**) based on the logical heritage of Avicenna and later logicians in such a way that we can show their relation with modern logical concepts.*
Keywords: *Avicennan logic, temporal modalities, time and modality.*

* * *

1. Introduction

Without any doubt, one of the brightest schools in the history of logic is "Avicennan logic". This school, in addition to devising independent logical ideas and innovations, is considered as an intermediate link in the transfer of Greek logical heritage to later periods. By "Avicennan logic" we are referring to a class of logical innovations whose foundations and essentials were devised by Avicenna (980-1037) and were later developed and completed by his followers, especially :

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